HARVARD Kennedy School
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Harvard Environmental Economics Program

February 2012
Discussion Paper 12-31

Unilateral Emission Tax and Intra-Industry Trade: An Ideal Variety Approach

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Acknowledgments

The Enel Endowment for Environmental Economics at Harvard University provides major support for HEEP. The Endowment was established in February 2007 by a generous capital gift from Enel, SpA, a progressive Italian corporation involved in energy production worldwide. HEEP is also funded in part by grants from the Alfred P. Sloan Foundation, the James M. and Cathleen D. Stone Foundation, Chevron Services Company, and Shell. HEEP enjoys an institutional home in and support from the Mossavar-Rahmani Center for Business and Government at the Harvard Kennedy School. HEEP collaborates closely with the Harvard University Center for the Environment (HUCE). The Center has provided generous material support, and a number of HUCE's Environmental Fellows and Visiting Scholars have made intellectual contributions to HEEP. HEEP and the closely-affiliated Harvard Project on Climate Agreements are grateful for additional support from the Belfer Center for Science and International Affairs at the Harvard Kennedy School, ClimateWorks Foundation, and Christopher P. Kaneb (Harvard AB 1990).

Citation Information

Clemenz, Gerhard. "Unilateral Emission Tax and Intra-Industry Trade: An Ideal Variety Approach," Discussion Paper 2012-31, Cambridge, Mass.: Harvard Environmental Economics Program, February 2012.

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Abstract: This paper compares total emissions of a uniformly mixing pollutant and welfare levels of a large country in autarky and with free trade with another large country that does not implement any environmental policies. There is intra-industry trade between the two countries which is modeled by using the ideal variety approach. Two abatement technologies are considered, a clean technology approach and an end-of-pipe approach. Emissions are influenced by an emission tax. With clean technology abatement emissions may be lower in the free trade regime than in autarky, with end-of-pipe abatement total emissions are greater with free trade. With both methods of abatement under free trade no emission tax may be levied at all if emissions per unit of output are very large, and if the gains from intra-industry trade due to an increase of available varieties are relatively small, autarky may yield a higher welfare level than free trade.

JEL-Classification: Q56, F18

Acknowledgements: I wish to thank Claude Clemenz, Wolfgang Rhomberg and Mike Scherer for helpful comments and suggestions.

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1. Introduction

In spite of mounting evidence pointing to a potentially dramatic global climate change with severe and possibly disastrous consequences for mankind, relatively little progress has been made with respect to designing and implementing internationally coordinated environmental policies to address the problem. There are various reasons for this sluggish response to a serious problem.

A still substantial though decreasing number of people flatly deny that global climate change takes place.

Others may accept its existence but deny that its consequences are nearly as bad as is claimed and/or refuse to accept that human activities have or can have a substantial influence on the global climate.

But even as the need for internationally coordinated environmental policies like the reduction of greenhouse gas emissions is now widely recognized, the process of reaching effective agreements has so far proved to be difficult and tedious. The Kyoto Protocol is still not ratified by all major countries, and the 2011United Nations Climate Change Conference in Durban was at best a very modest success. It was agreed that at the latest by 2015 a universal legal agreement on climate change that would include non-Annex I countries should be reached which would become effective by 2020. However there is concern that the commitments to cut emissions may be insufficient to keep the rise in global average temperatures below two degrees Celsius.

It is not the purpose of this paper to investigate why it is so hard to reach effective international agreements on this issue, but it seems fair to say that the degree of environmental concerns varies considerably from country to country. The big question for countries which are particularly concerned about the climate change is what to do in the absence of international agreements, in particular, whether it makes sense to implement national policies unilaterally. One widespread view on this issue is that unilateral environmental policies are harmful for the country implementing them without much mitigating the problem of climate change. Environmental policies are costly, affect the competitiveness of the domestic firms and lead to a shift of production to countries with no or less stringent environmental regulations.

Interestingly, this view is shared by many environmentalists who, however, draw rather different conclusions with respect to what should be done. Whereas one group uses this argument to prevent environmental policies altogether, the other group advocates a restriction of international trade and economic globalization.

The purpose of this paper is to look at the merits of these arguments from a theoretical point of view. Assuming two big countries with different attitudes towards environmental issues implying that only one of them – we refer to it as the home country – is prepared to implement environmental policies, the following questions are addressed.

i. Are there conditions ensuring that free trade yields higher (lower) welfare for the home country than autarky?
ii. Are there conditions under which an emission tax imposed only in the home country increases (reduces) domestic welfare in a free trade regime?
iii. Are there conditions where the optimal domestic emission tax leads to more (fewer) emissions under free trade than in autarky?

These questions are analyzed for intra-industry trade, using the ideal variety approach of markets for a differentiated product. The starting point is Salop's extension of the Hoteling model of spatial differentiation (Salop 1979), in which it is assumed that each consumer favors one – her ideal – variety, and her utility is decreasing in the distance between this ideal variety and the one actually bought. The only source for gains from international trade is an increase in the number of available varieties and consequently a reduction of the expected disutility of not getting the ideal variety. The marginal disutility of a divergence between the ideal and the consumed variety provides a convenient measure for the market power of firms each of which is producing one variety, and also for the potential gains from trade.

The paper will show below that in this framework the answers to the above questions depend on the type of abatement technology, on marginal abatement costs, marginal emissions and the strength of the preference for the ideal variety. As far as abatement is concerned, two alternatives are considered: Clean technology and end–of–pipe abatement. Clean technology requires an investment up-front in order to reduce emissions per unit of output, while, by assumption, marginal costs of production are not affected. End–of–pipe abatement requires an increase of marginal costs of production in order to reduce emissions. Pollutants are assumed to be uniformly mixing, a feature of greenhouse gases, hence a shift of production from the environmental friendly domestic country to the foreign country has a negative effect on the home country as total emissions increase.

In the clean technology version, the paper shows that free trade is superior to autarky for large potential gains from trade, i.e. if individual preferences for the ideal variety are strong. If emissions per unit of output are not very large relative to these preferences, in particular, if full abatement does not violate the non-negativity constraint for the profits of domestic firms, then free trade induces higher abatement levels than would be optimal in autarky. If emissions are too large for making full abatement feasible, however, abatement is less than in autarky, and if this is combined with small potential gains from trade, autarky may be better than free trade.

If an end‒of‒pipe technology is assumed, it appears that free trade is better than autarky if gains from trade excluding environmental aspects are sufficiently large, though the critical value for the preference for the ideal variety is much larger than for clean technology abatement. On the other hand, in this end-of-pipe technology case domestic emission taxes are beneficial for much larger emissions per unit of output than with a clean technology. However, total emission reductions are always smaller under free trade than in autarky. The reason is that end‒of‒ pipe abatement increases marginal cost of production and reduces the domestic share of total output.

The plan of the paper is as follows: In the next section we review briefly some of the related literature. This is followed by a description of the Salop model, which we extend to include emissions. Then we turn to the clean technology approach and derive the optimal emission tax under autarky. The same exercise is performed for a free trade regime in which only one country is ready to impose an emission tax. This is followed by a comparison of emissions per firm, total emissions and the welfare levels in autarky and under free trade. Then we repeat the analysis for the end‒of‒pipe approach. The conclusion contains a summary of results as well as their possible policy implications and suggestions for future research.

2. Related Literature

The importance of international commodity and factor movements for the effectiveness of environmental policies was recognized right from the beginning of the establishment of environmental economics as a special branch of economics, with Baumol (1971) as an early example. Given the state of international economics at the time, it is not surprising that the focus was on trade under conditions of perfect competition. Beginning in the early nineties, however, as trade models with imperfect competition were well established (see Helpman and Krugman 1985), they were also used more frequently to analyze international aspects of environmental issues. A comprehensive survey of the literature on trade and environment can be found in Rauscher (2006), who notes that only a few contributions focus on intra-industry trade, even though it accounts for around 50% of total commodity trade. Explanations for intra-industry trade are either based on oligopoly models with firms located in different countries and producing a homogenous product (Brander 1981, Brander and Krugman 1983), or on models of differentiated products in which different varieties of a good are traded between countries. As this paper uses the latter approach we refer the reader to Neary (2006) and Rauscher (2006) for surveys of the main contributions and results of the former brand of models.

Models of intra-industry trade with differentiated goods and the environment usually employ the "love of variety approach" which is based on the seminal paper by Dixit and Stiglitz (1977) and has been introduced to trade theory by Krugman (1979, 1980). To the best of our knowledge, the "ideal variety approach" due to Hotelling (1927) and Salop (1979), which was used by Helpman (1981) to analyze intra-industry trade, has not been applied to international aspects of environmental policies.

Using the love of variety approach, Rauscher (1997) has shown that the effect on the environment of moving from autarky to free trade depends on the change in the number of firms, which may be positive or negative, depending on the specification of the utility function of the representative consumer. Pollution is reduced if environmental regulation aims at keeping emissions per firm constant and there is market exit. Considering leakage effects, Gürtzgen and Rauscher (2000) show that, within this framework, tighter regulations in the home country may also reduce emissions in the foreign country. Pflüger (2001) analyzes the efficiency of environmental taxes when firms relocate between countries with different environmental standards, but in contrast to the previous papers, the number of firms is assumed to be constant. Neary (2006) extends this analysis to allow for a large number of industries with different emission levels and finds that the relocation effects are far less dramatic than suggested by a one-sector model.

An interesting variation of the differentiated good approach has been offered by Benarroch and Weder (2006), who consider intra-industry trade in intermediate products. They assume that differentiated intermediate goods are required for the production of the final output, and these inputs differ not only with respect to their characteristics but also as far as pollution in the production of the final good is concerned. Emission taxes are determined by the two countries in a non-cooperative game. Depending on the emission functions, total emissions may be larger or smaller under free trade than in autarky.

The paper by Haupt (2006) is most closely related to the present paper, as he considers different abatement technologies and their impact on environmental policies. His main focus, however, is the effect of environmental policies on the total number of firms and varieties. Assuming that the two countries set emission standards in a non-cooperative way, he finds that policies are too restrictive as compared to a first best solution, whereas a comparison of autarky and free trade is ambiguous as far as total pollution is concerned.

The present paper differs from the literature in various aspects. It uses an “ideal variety approach” for modeling intra-industry trade, and it assumes a strong asymmetry with respect to the environmental concerns of the two countries. The main focus is on emission taxes and not on standards, though the latter can easily be dealt with. Finally, the number of firms is exogenous, thus ruling out effects which are central to the contributions of Benarroch and Weder (2006) and Haupt (2006).

We now turn to the ideal variety approach and its implications for one–sided emissions taxes when moving from autarky to intra-industry trade.

3. The Ideal Variety Approach

The workhorse for our analysis is the standard Salop-model (Salop 1979) with n firms located equi-spaced around a circle with circumference equal to 1 and a continuum of consumers with measure η uniformly distributed around the circle. Each point on the circle corresponds to a variety of a differentiated product. Each consumer buys at most one unit of one variety, and her utility depends on the intrinsic utility of the good which is denoted as β and assumed to be the same for all consumers and all varieties, the price p of the variety and the distance between the location of the consumer and that of the variety. Each firm produces exactly one variety, and for simplicity the marginal costs are assumed to be equal to zero. The utility of consumer j when buying one unit of variety i at price p_i equals

$$u_j(p_i) = \beta - p_i - \delta \Delta_{ij} \quad (1)$$

where δ denotes the constant marginal disutility of the difference between a consumer's location (i.e. her "ideal variety") and the location of the variety bought, and Δ_{ij} denotes this distance.

It is well known that in equilibrium each firm charges a price $p^* = \delta/n$ and sells η/n units of its product. Introducing fixed costs and assuming free entry it can be shown that in equilibrium the endogenous number of firms is larger than in a social optimum, but for the purpose of this paper we assume that n is exogenous. Next we consider a negative production externality and the effect on the equilibrium if an emission tax is introduced and firms can invest in abatement.

4. Clean Technology Abatement

4.1 Autarky

We assume that the production of one unit of the differentiated product generates a negative externality denoted as *e* and evaluated in monetary terms. Each firm can invest in a cleaner technology which reduces the emission *e* by an amount of *r*. In order to do so it has to spend an amount of $F(r)$ where *F* is strictly convex and increasing in *r*. To provide an incentive for firms to invest in the cleaner technology the government introduces an emission tax with the constant marginal tax rate τ. After τ has been announced the firms play the following two–stage game:

Stage 1: Each firm invests in a cleaner technology by choosing the emission reduction level *r*.

Stage 2: Firms compete via prices.

Solving the game backwards we start with stage 2 and consider two adjacent firms labeled *i* and *j* who compete for the consumers located between them. It is well known that the share of the two firms in the market segment between them equals

$$s_i = \frac{p_j - p_i + \delta/n}{2\delta} \tag{2}$$

For given levels of r_i and r_j each firm chooses its price in order to maximize its profit:

$$\pi_i = [p_i - \tau(e - r_i)]\eta s_i \tag{3}$$

The corresponding first order conditions are

$$2p_i - p_j - \tau(e - r_i) - \delta/n = 0 \tag{4a}$$

$$2p_j - p_i - \tau(e - r_j) - \delta/n = 0 \tag{4b}$$

implying

$$p_i - p_j = \tau(r_j - r_i)/3 \tag{5}$$

From (4a) and (4b) we get the equilibrium prices for given r_i and r_j.

$$p_i(r_i, r_j) = \delta/n + \tau e - \tau(2r_i + r_j)/3 \tag{6}$$

Plugging (5) and (6) back into the profit function (3) and assuming symmetry between all firms we get for the objective function of firm i in stage 1

$$\pi_i = \frac{\eta}{\delta}\left[\frac{\delta}{n} + \frac{\tau(r_i - r_j)}{3}\right]^2 - F(r_i) \tag{7}$$

implying the first order condition

$$\frac{\partial \pi_i}{\partial r_i} = \frac{2\tau\eta}{3\delta}\left[\frac{\delta}{n} + \frac{\tau(r_i - r_j)}{3}\right] - F'(r_i) = 0 \tag{8}$$

The second order condition must be negative, hence by the implicit function theorem we get

$$sign\frac{\partial r_i}{\partial r_j} = sign\frac{-2\eta\tau^2}{9\delta} < 0 \tag{9}$$

$$sign\frac{\partial r_i}{\partial \eta} = sign\frac{2\tau}{3\delta}\left[\frac{\delta}{n} + \frac{\tau(r_i - r_j)}{3}\right] \tag{10}$$

$$sign\frac{\partial r_i}{\partial \tau} = sign\frac{2\eta}{3\delta}\left[\frac{\delta}{n} + \frac{2\tau(r_i - r_j)}{3}\right] \tag{11}$$

$$sign\frac{\partial r_i}{\partial n} = sign\frac{-2\tau\eta}{n^2} < 0 \tag{12}$$

According to (9) the levels of investments in cleaner technologies are strategic substitutes, meaning that the marginal return on such an investment is decreasing in the abatement level of the competitor. The signs of (10) and (11) are ambiguous, investment of firm *i* in a cleaner technology is increasing in the market size and in the marginal tax rate of the emission tax if her abatement is already greater than that of her rival or if the difference is small. Finally, according to (12) the investment in abatement of each firm is decreasing in the total number of firms.

Solving (8) for a symmetric equilibrium with $r_i = r_j = r_a$ we get

$$\frac{2\tau\eta}{3n} = F'(r_a) \tag{13}$$

In order to obtain the social optimum for given n a social planner would minimize the sum of abatement costs and total damages caused by emissions, hence solving for r

$$\min\{\eta(e - r) + nF(r)\}$$

implying

$$F'(r) = \eta/n \tag{14}$$

In order to achieve the socially efficient abatement and pollution level we therefore substitute (14) for $F'(r)$ in (13) in order to obtain the socially optimal emission tax rate τ_a:

$$\tau_a = 3/2 \tag{15}$$

In order to keep the analysis tractable we shall assume in later section that $F(r) = r^2/2$, implying $F'(r) = r$. Assuming that the rest of the world is producing total emissions equal to e and that $e \geq 1/n$ we get for the optimal level of domestic welfare in autarky

$$W_a = \beta - 2e + 1/2n - \delta/4n \tag{16}$$

It is noteworthy that the optimal tax rate is greater than the marginal damage of emissions, which is due to the market power of firms who can pass on part of the emission tax on consumers.

4.2 Free Trade

4.2.1 Equilibrium

We consider next the model with free trade in the differentiated product by two countries which are identical except for their environmental policy. Specifically, we assume that only the home country levies an emission tax τ per unit of emission. In order to save notation we set η equal to 1 for each country. Finally, we assume that firms locate in the product space such that each domestic firm is located between two foreign firms and vice versa. The distance between any two firms equals $1/2n$. Subscript h refers to the home country, subscript f to the foreign country. The main differences to the autarky model above are that the number of firms is now $2n$, the number of consumers has doubled, and only the domestic firms are subject to the emission tax. Consequently the profit functions for a domestic firm with abatement level r_h and the adjacent foreign firm can be written as

$$\pi_h = [p_h - \tau(e - r_h)] \frac{p_f - p_h + \delta/2n}{\delta} \tag{17}$$

$$\pi_f = p_f \frac{p_h - p_f + \delta/2n}{\delta} \tag{18}$$

The associated first order conditions are

$$\frac{\partial \pi_h}{\partial p_h} = -2p_h + p_f + \frac{\delta}{2n} + \tau(e - r_h) = 0 \tag{19}$$

$$\frac{\partial \pi_f}{\partial p_f} = p_h - 2p_f + \frac{\delta}{2n} = 0 \tag{20}$$

Solving (19) and (20) for p_h and p_f yields

$$p_h = \frac{\delta}{2n} + \frac{2\tau(e - r_h)}{3} \tag{21}$$

$$p_f = \frac{\delta}{2n} + \frac{\tau(e - r_h)}{3} \tag{22}$$

implying

$$p_h - p_f = \frac{\tau(e - r_h)}{3} \tag{23}$$

A first comparison between autarky and free trade prices shows that there are various effects pulling in different directions. The larger number of competing firms has a dampening effect on prices in both countries as $\delta/2n < \delta/n$. As far as the domestic country is concerned a smaller fraction of the emission tax is passed on to consumers. In the foreign country, on the other hand, the taxation of domestic firms softens price competition for foreign firms and prices are higher than they would be without taxation in either country. A full comparison of the free trade and the autarky equilibrium requires taking account of the changes in the abatement levels and of the optimal tax rate. We turn to the first issue and solve the first stage of the free trade game. Using (21) – (23), the profit function of a domestic firm with two identical foreign firms on each side can be written as

$$\pi_h = \frac{2}{\delta}\left[\frac{\delta}{2n} - \frac{\tau(e - r_h)}{3}\right]^2 - F(r_h) \tag{24}$$

The first order condition equals

$$\frac{\partial \pi_h}{\partial r} = \frac{4\tau}{3\delta}\left[\frac{\delta}{2n} - \frac{\tau(e-r_h)}{3}\right] - F'(r_h) = 0 \qquad (25)$$

Comparing (25) and (13) yields immediately the following result.

Proposition 1: For a given emission tax rate τ a move from autarky to free trade with a country without emission tax leads to a reduction of the domestic abatement level.

Proof:

$$F'(r_a) = \frac{2\tau}{3n} > \frac{2\tau}{3n} - \frac{4\tau^2(e-r_h)}{9\delta} = F'(r_h) \qquad (26)$$

and $r_a > r_h$ follows from the strict convexity of $F(r)$. Q.E.D.

Note that proposition 1 follows already from the fact that abatement levels are strategic substitutes (see equation (9)) as paying no emission tax is formally equivalent to $r_f = e$ with respect to price competition.

From (25) we also get

$$sign\frac{\partial r}{\partial \delta} = sign\frac{4\tau^2(e-r_h)}{9\delta^2} > 0$$

In order to keep the analysis tractable we shall use from now on the quadratic abatement cost function introduced earlier, i.e. $F(r) = r^2/2$. Substituting this in (25) yields

$$r(\tau) = \frac{2\tau(3\delta - 2\tau en)}{n(9\delta - 4\tau^2)} \qquad (27)$$

implying

$$r'(\tau) = \frac{6\delta(9\delta + 4\tau^2 - 12\tau en)}{n(9\delta - 4\tau^2)^2} \tag{28}$$

and

$$r'(0) = 2/3n > 0 \tag{29}$$

Note that (28) is only well defined for $9\delta > 4\tau^2$, and we shall assume that this is the case unless stated otherwise. Furthermore

$$\frac{\partial r}{\partial e} = -\frac{4\tau^2 n}{n(9\delta - 4\tau^2)} < 0 \tag{30}$$

4.3 Optimum Emission Tax

4.3.1 The Domestic Welfare Function

Domestic welfare in the free trade regime, denoted as W_h has four components: The first is the intrinsic utility of the differentiated product, denoted as β. Since by assumption each consumer consumes one unit in equilibrium it is not affected by switching from autarky to free trade. Then there is the average disutility due to the distance between ideal and consumed variety, denoted as D_k, $k = a,h$, where subscript a refers to autarky and h to free trade. This is smaller under free trade as the number of varieties doubles, the maximum difference between D_a and D_h being $\delta/8n$. Since the market share of domestic firms is smaller than 1/2 unless we have $r_h = e$ or $\tau = 0$ the welfare gain due to free trade $\Delta D = D_h - D_a$ is less than $\delta/8n$. A further disutility is caused by the costs of pollution including abatement costs, and we denote it as $E_h = 2e - r_h x_h - F(r_h)$. Finally, we have to consider the balance of trade, denoted as $B = [p_h x_h - p_f x_f]/2$. Note that D, E and B are all functions of the emission tax rate τ. Collecting terms we get

$W_h = \beta - D_h - E_h + B$ (31)

and the task of the domestic policy maker is to maximize W_h with respect to τ.

4.3.2 Brand Preference

Looking at (31) in more detail we get for D_h

$$D_h = \delta n\left[\frac{1}{8n^2} + \frac{\tau^2\left(e - r_h(\tau)\right)^2}{18\delta^2}\right] = \frac{\delta}{8n} + \frac{n\tau^2\left(e - r_h(\tau)\right)^2}{18\delta} \tag{32}$$

implying

$$\frac{\partial D_h}{\partial \tau} = \frac{n\tau\left(e - r_h(\tau)\right)\left[e - r_h(\tau) - \tau r_h'(\tau)\right]}{9\delta} \tag{33}$$

which equals zero for $\tau = 0$. In fact, D_h reaches its (local) minimum at $\tau = 0$ where its second derivative with respect to τ is strictly positive. For $\tau = 0$ as well as for τ^e satisfying $e = r(\tau^e)$ total disutility due to differences between ideal and actual varieties reaches a minimum, and conversely its maximum at τ for which $e - \tau - \tau r'(\tau) = 0$ holds. In fact $-D_h$ is a strictly convex function in the relevant range, i.e. for $0 \leq \tau \leq \tau^e$.

4.3.3 Environmental Costs

Total environmental costs E_h including abatement costs equal

$$E_h(\tau, e) = 2e - r(\tau)\left[1 - \frac{2n\tau\left(e - r(\tau)\right)}{3\delta}\right] + nF\left(r(\tau)\right) \tag{34}$$

where the expression in square brackets denotes total domestic production which equals 1 if either $\tau = 0$ or $r = e$.

The first derivative of E_h with respect to τ equals

$$\frac{\partial E_h}{\partial \tau} = -\frac{1}{3\delta}\{r'(\tau)[3\delta(1-nF'(r(\tau)))-2n\tau(e-2r(\tau))]-2nr(\tau)(e-r(\tau))\}$$

$$= -r'(\tau)\left[1 - nF'(r(\tau)) - \frac{2n\tau(e-r(\tau))}{3\delta}\right] + \frac{2nr(\tau)(e-r(\tau)-\tau r'(\tau))}{3\delta} \qquad (35)$$

For the sake of tractability assume again $F(r) = r^2/2$. For $\tau = 0$ we get

$$\frac{\partial E_h}{\partial \tau|_{\tau=0}} = \frac{2}{3n} \qquad (36)$$

Finally, we can also calculate τ^e explicitly:

Lemma: $\tau^e = 3en/2$. (37)

Proof:

$$e = \frac{2\tau(3\delta - 2\tau en)}{n(9\delta - 4\tau^2)}$$

Substituting τ^e then yields

$e(9\delta - 9e^2n^2)/(9\delta - 9e^2n^2) = e.$ Q.E.D.

4.3.4 Balance of Trade

Finally, for the balance of trade we get

$B_h = [p_hx_h - p_fx_f]/2.$

which can be written as

$$B_h = -\frac{n\tau\left(e - r_h(\tau)\right)}{3\delta}\left[\frac{\delta}{2n} + \tau\left(e - r_h(\tau)\right)\right] \tag{38}$$

$$\frac{\partial B_h}{\partial \tau} = -\frac{n}{3\delta}\left[\frac{\delta}{2n} + 2\tau\left(e - r(\tau)\right)\right][e - r(\tau) - \tau r'(\tau)] \tag{39}$$

For $F(r) = r^2/2$ we get

$$\frac{\partial B_h}{\partial \tau|_{\tau=0}} = -\frac{e}{6} \tag{40}$$

4.3.5 Social Optimum

A main difficulty is that in general $W(\tau)$ is not (quasi)-concave, though it is for some parameter configurations in the relevant range. Consequently, the usual necessary conditions for a maximum have to be considered with care. Before turning to this issue in more detail we have to state the constraints of the problem. First we have non-negativity constraints on r and τ, hence

$$r,\ \tau \geq 0. \tag{41}$$

Secondly abatement per unit of output cannot exceed emissions, hence

$$r \leq e. \tag{42}$$

Finally there is a participation constraint as firms will stop production if their gross profits do not cover the investment in the cleaner technology. Using the quadratic cost function we get

$$\frac{r(\tau)^2}{2} \leq \frac{2}{\delta}\left[\frac{\delta}{2n} - \frac{\tau\big(e - r(\tau)\big)}{3}\right]^2$$

or

$$r(\tau) \leq \frac{\delta^{1/2}}{n} - \frac{2\tau\big(e - r(\tau)\big)}{3\delta^{1/2}} \tag{43}$$

which yields after simple manipulations

$$r \leq \frac{\frac{3\delta}{n} - 2\tau e}{3\sqrt{\delta} - 2\tau} \tag{43'}$$

As domestic profits in the free trade equilibrium are smaller than in autarky even without payment of emission taxes, this is more restrictive than the participation constraint in autarky.

4.4 Welfare Comparisons

Next we turn to the problem of a comparison of domestic welfare in autarky and with free trade if the foreign country does not impose environmental policies. In the first step we look at the first derivative of W_h with respect to τ. Using (33), (35) and (39) and rearranging we get for $F(r) = r^2/2$

$$\frac{\partial W_h}{\partial \tau} =$$

$$= r'(\tau)\left[1 - nr(\tau) - \frac{2n\tau\big(e - r(\tau)\big)}{3\delta}\right] - \big(e - r(\tau) - \tau r'(\tau)\big)\left[\frac{1}{6} + \frac{7n\tau\big(e - r(\tau)\big)}{9\delta} + \frac{2nr(\tau)}{3\delta}\right] \tag{44}$$

At first we establish conditions under which there exists an emission tax which increases domestic welfare as compared to no environmental policies.

Proposition 2: In the free trade regime there exists an environmental tax τ such that domestic welfare is larger than without tax if and only if $e < 4/n$.

Proof: Substituting $\tau = 0$ in (44) yields

$$\frac{\partial W_h}{\partial \tau|_{\tau=0}} = \frac{2}{3n} - \frac{e}{6} \tag{45}$$

which is greater than zero for $e < 4/n$, which proves the if-part.

Next we show that $e > 4/\mathrm{n}$ implies that $\partial W/\partial \tau < 0$ for all $\tau \geq 0$ within the relevant range. Full abatement cannot be optimal for $e > 4/n$ as total costs exceed benefits for $e > 2/n$. An interior maximum of $W(\tau)$ requires (44) to equal zero which in turn requires $r < 1/n$. But for $r < 1/n$ the second term of (44) remains strictly negative, hence $W'(\tau) < 0$ continues to hold. Q.E.D.

From an environmentalist point of view the negative message of this proposition is that in a free trade regime an emission tax is not warranted when it is needed most, that is when emissions per unit of output are particularly high. The reason is that with high costs of investing in a cleaner technology abatement is relatively low and the tax that has to be paid for the remaining emissions becomes very costly in terms of market shares and the trade balance within the market for the differentiated product. This does not mean, however, that free trade is not preferable to autarky even in this case. The next proposition shows that free trade is superior to autarky for $\delta > 4$ if the optimal environmental tax is imposed in both regimes.

Proposition 3: If $\delta > 4$ then there exists a free trade equilibrium with greater domestic social welfare than is attainable in autarky.

Proof: Recall that without an environmental tax total disutility in autarky minus total disutility under free trade equals $\delta/8n$. The efficient level of abatement in autarky is $1/n$ implying total environmental costs of $2e - 1/2n$, and $\delta/8n \geq 1/2n$ if $\delta \geq 4$. Q.E.D.

The source of gains from free trade in the differentiated product market is an increase in the number of varieties offered which is the more valuable the stronger the preferences of consumers for their ideal variety are. Consequently, for δ sufficiently large, increased consumer satisfaction due to a greater number of available varieties will compensate even for a complete abandonment of any abatement. However, there is a downside to this: If δ is not that large and emissions per unit of output are substantial such that no welfare improving emissions tax exists in the free trade regime, then the latter may yield less social welfare than autarky with efficient taxation. This is summarized in the following Proposition.

Proposition 4: If $\delta < 4$ and $e \geq 4/n$ then free trade is inferior to autarky.

Proof: Follows immediately from propositions 2 and 3. Q.E.D.

Finally we turn to the intermediate case with $\delta < 4$ and $e < 4/n$. We already know from proposition 2 that in this case also in the free trade regime a welfare increasing emission tax exists. We now show that under certain conditions free trade is better than autarky and that abatement may be higher, though at an inefficiently high level. To see this consider first a domestic policy aimed at complete abatement, i.e. $r = e$. Domestic welfare with free trade and complete abatement is denoted as W_{dc} and given by

$$W_{dc} = \beta - e - ne^2/2 - \delta/8n \qquad (46)$$

We get

Proposition 5: W_a <(>) W_{dc} if e < (>) $1/n + \delta^{1/2}/2n$.

Proof: For $r(\tau) = e > 0$ we get

$W_a - W_{dc} = 1/2n - \delta/4n - e + ne^2/2 + \delta/8n = 0$ if $4n^2e^2 - 8ne + 4 - \delta = 0$,

which holds for e satisfying

$$e = \frac{8n \mp \sqrt{64n^2 - 64n^2 + 16\delta n^2}}{8n^2} = \frac{1}{n} + \frac{\delta^{1/2}}{2n}$$

Q.E.D.

An immediate consequence of proposition 5 is that domestic welfare attainable under free trade is higher than in autarky if full abatement is feasible (i.e. $e \leq \delta^{1/2}/n$) and $\delta \leq 4$, as the following corollary shows.

Corollary 1: Let $e \leq \delta^{1/2}/n$ and $\delta \leq 4$ then $W_a \leq W_{dc}$.

Proof: $\delta^{1/2}/n \leq 1/n + \delta^{1/2}/2n$ if and only if $\delta \leq 4$: Multiplying both sides of the inequality by $2n$ yields $\delta^{1/2} \leq 2$ which holds as an equality for $\delta = 4$. Q.E.D.

But even if $\delta = 4$ and $e = 2/n$, the highest attainable welfare level with free trade is strictly better than that achievable in autarky as the next corollary shows.

Corollary 2: Full abatement is not optimal for $\delta = 4$ and $e = \delta^{1/2}/n$ and the optimal emissions tax rate yields a higher utility level than in autarky.

Proof: $\delta = 4$ and $e = 2/n$ implies $W_a = W_{dc} = W_h(0)$. According to proposition 2 $W_h'(0) > 0$, hence there exists an optimal tax rate $\tau > 0$ which ensures higher domestic welfare than in autarky.

Q.E.D.

This result is illustrated in Figure 1 for $\delta = 4$, $n = 3$ and $e = 0.66$. The horizontal line represents the welfare level with full abatement, the curved line is the graph of $W_h(\tau)$. Note that 0.66 < $\delta^{1/2}/3$, and τ_e equals 2.97.

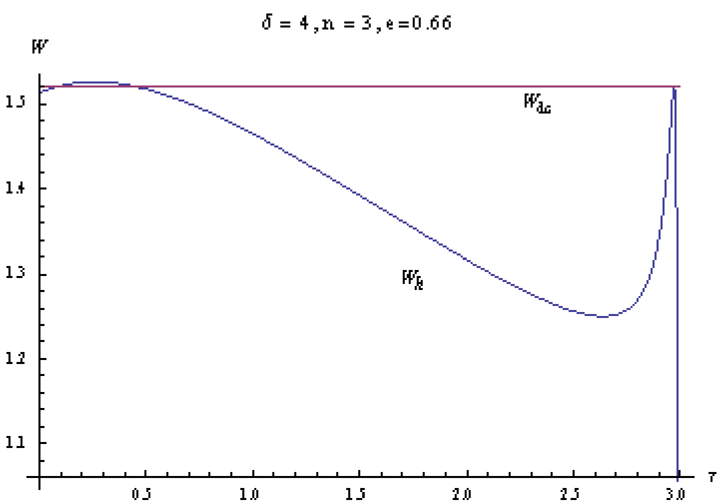

Figure 1

As can be seen from Figure 1 the maximum of $W_h(\tau)$ is an interior point even though e is slightly smaller than $\delta^{1/2}$/n. As can also be seen $W_h'(\tau)$ is strictly positive as $\tau \to \tau_e$ and thus $r(\tau) \to e$. Setting r equal to e in (44) yields

$$\lim W_h'(r(\tau) \to e) = nr'(\tau)[1/n - e + \tau/6 + 2e/3\delta] \quad (46)$$

Substituting $\delta^{1/2}/n$ for e and $3en/2$ for τ we get

$$\lim W_h'(r(\tau) \rightarrow e) = nr'(\tau)[1/n - \delta^{1/2}/n + \delta^{1/2}/4 + 2/3\delta^{1/2}] \quad (47)$$

which is strictly positive for $1 \le \delta \le 4$. Consequently $W_h(\tau)$ is not concave and not even quasi-concave for e sufficiently close to $\delta^{1/2}/n$. In fact, it is not well defined for $e = \delta^{1/2}/n$ and $\delta = 4$ at $\tau = 3en/2$ as the denominator equals zero. Fortunately, corollary 2 ensures that we don't need to worry about that as, in this borderline case, we get an interior maximum of W_h.

The reason why $W_h'(\tau)$ may change its sign more than once is illustrated in Figure 2. Recall that $W_h(\tau) = \beta - E(\tau) - D(\tau) - B(\tau)$, and as can be seen in Figure 2 (and is easily checked) in the relevant rage $-E(\tau)$ is concave whereas $-D(\tau)$ and $B(\tau)$ are convex, both reaching their minimum at τ satisfying $e - r(\tau) - \tau r'(\tau) = 0$. For small τ the marginal reduction of total emission costs dominates the adverse effect on D and B. As τ is increased further the reduction of E approaches zero whereas D and B continue to reduce W_h further. As soon as D and $-B$ reach their peak a further increase of τ improves the balance of trade, and at some point this effect dominates the increase of E. For smaller values of e, D and B start to improve before E reaches a minimum and in the relevant range $W_h(\tau)$ is (quasi)–concave (see Figure 4 below).

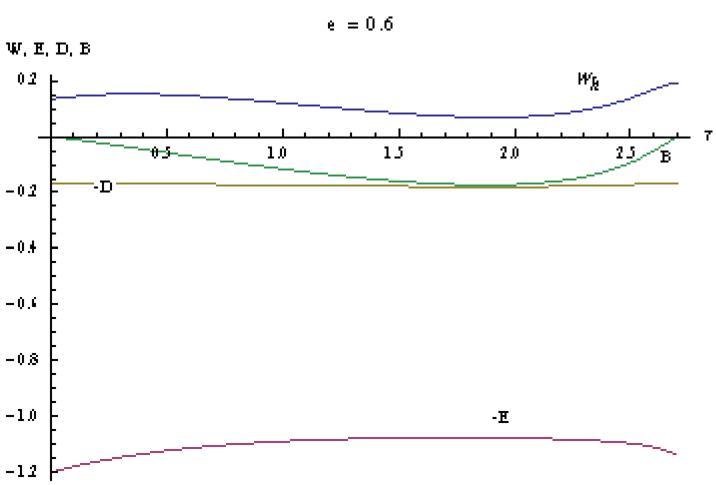

Figure 2

In Figure 2 the curve on the bottom is $-E(\tau)$, the curve above it is $-D(\tau)$ which reaches its minimum at the same τ as $B(\tau)$ which is smaller than zero in the relevant range. In the middle range of τ its marginal effect on E is small because it increases r, but at the same time the marginal costs of r are also increasing, and in addition domestic production goes down thus reducing the beneficial effect of increasing τ even further. To the right of the minimum of $B(\tau)$ the latter effect is reversed and $W_h(\tau)$ is again increasing in τ until $r(\tau) = e$.

While the shape of $W_h(\tau)$ for $e = 0.6$ is similar to that for $e = 0.66$ and an interior local maximum of $W_h(\tau)$ still exists it is no longer greater than the full abatement welfare level W_{dc}, as is shown in Figure 3.

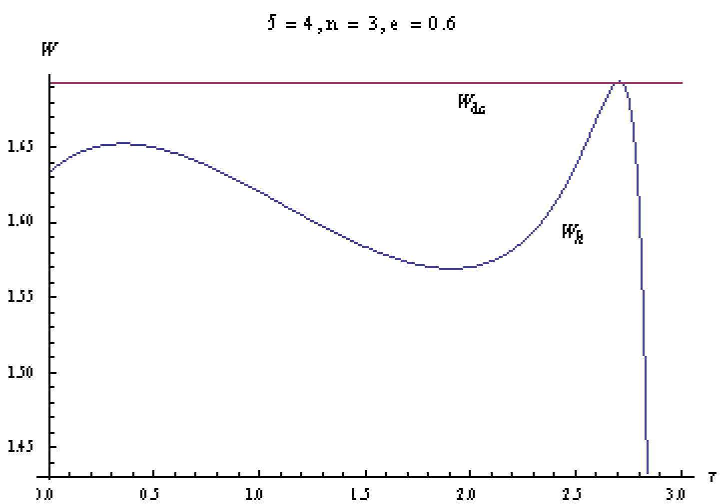

Figure 3

It is shown below that $\partial W_{dc}/\partial e < \partial W_{di}/\partial e$, where W_{di} denotes the welfare level of an interior local maximum. In fact, for e sufficiently close to $1/n$ an interior local maximum ceases to exist and W_h is strictly increasing in τ implying that full abatement is optimal. This is illustrated in Figure 4.

As can be seen for e = 0.4 > 1/3 domestic welfare is strictly increasing in τ until $\tau = \tau_e$ = 1.8 and $r(\tau) = e$. Figure 5 illustrates where this change of the shape of $W_h(\tau)$ comes from by drawing a diagram which is analogous to Figure 2. Note however, that in the latter τ_e equals 2.7, and in addition we have changed β from 2 to 1.5 in order to keep the diagram small.

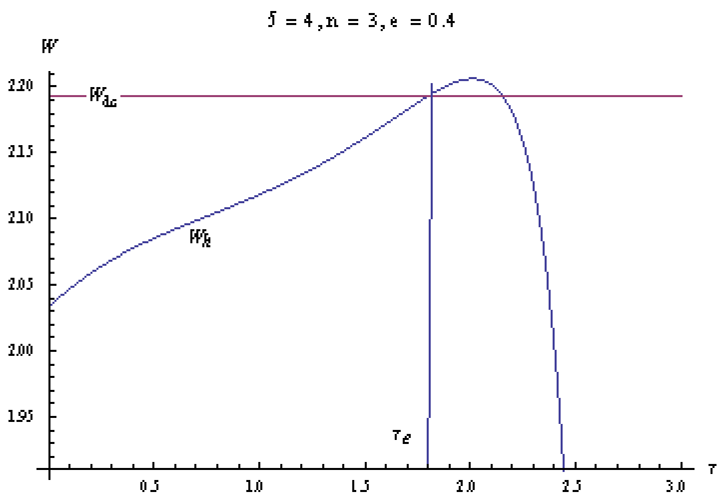

Figure 4

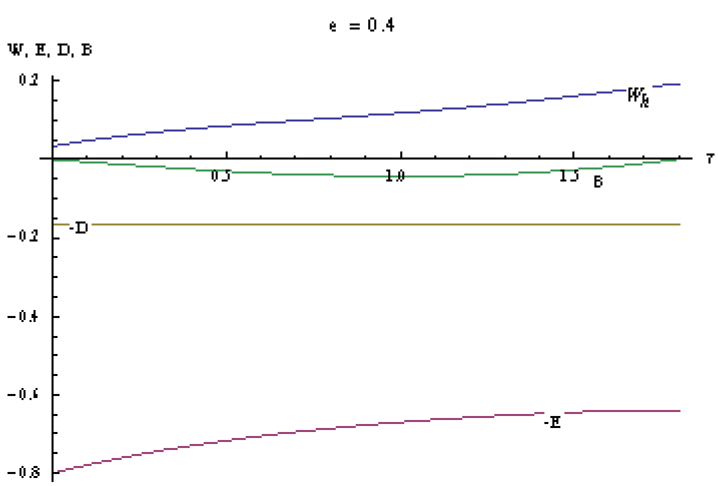

Figure 5

As can be seen from Figure 5 $-E_h'(\tau)$ is positive over the relevant range, and the balance of trade effect is rather small. Consequently, the adverse effect on B of increasing τ is weaker than the positive effect of reducing E. At some point increasing τ improves B and the overall effect on domestic welfare remains positive. In any case, we have established for $\delta \leq 4$ and $e \leq \delta^{1/2}/n$ that imposing an emission tax only in the domestic country increases its welfare which is greater than in autarky, and in addition also total abatement may be greater. In fact, less than full abatement is optimal only in a small neighborhood of $\delta = 4$ and $e = 2/n$. Before turning to this result we state as a Proposition a useful property of an interior local maximum of W_h which is already obvious from Figures 1 and 3.

Proposition 6: Suppose $1 \leq \delta \leq 4$ and $0 < e \leq 4/n$. If an interior maximum of $W_h(\tau)$ with $r(\tau) < e$ exists then $r(\tau) < 1/n$.

Proof: This follows immediately from setting $\partial W_h/\partial\tau$ equal to zero in (44). For small values of τ the first expression in square brackets on the r.h.s. is only positive as long as $r < 1/n$. For $r > 1/n$ this expression is negative and when $\partial W_h/\partial\tau = 0$ again it is at a local minimum. Q.E.D.

Figures 1 and 3 suggest that for $\delta = 4$ and $e = 2/n$ a relative small reduction of e implies a switch of the optimal policy from the interior maximum with little abatement to full abatement. This is confirmed by the following result.

Proposition 7: If $\delta = 4$ and e = $2/n$ the following inequality holds.

$\partial W_{dc}/\partial e < \partial W_{di}/\partial e < 0$.

Proof:

$\partial W_{dc}/\partial e = -1 - ne = -3$ for $e = 2/n$.

Fort the interior maximum we apply the envelope theorem in order to obtain

$$\frac{\partial W_{di}}{\partial e} = -2 - \frac{2n\tau}{3\delta}\left[r(\tau) + \frac{7\tau\big(e - r(\tau)\big)}{3}\right]$$

Substituting for δ and e we get

$$\frac{\partial W_{dc}}{\partial e} - \frac{\partial W_{di}}{\partial e} = \frac{2n\tau}{3\delta}\left[r(\tau) + \frac{14\tau}{3n} - \frac{7r(\tau)}{3} - \frac{6}{n\tau}\right] < 0$$

since $\tau < 1$ at an interior maximum. Q.E.D.

A straightforward corollary of proposition 7 states that assuming $e = \delta^{1/2}/n$ and decreasing δ – starting with $\delta = 4$ – increases the welfare level of an interior local maximum by less than the full abatement level, hence there exists a critical level of $\delta < 4$ such that full abatement is optimal for any δ smaller than this critical level.

Corollary 3: Assume $e = \delta^{1/2}/n$. At $\delta = 4$ we get

$\partial W_{dc}/\partial\delta < \partial W_{di}/\partial\delta < 0$.

Proof: Note first that $\partial e/\partial\delta = 1/2n\delta^{1/2}$. Consequently, we get

$$\frac{\partial W_{dc}}{\partial \delta} - \frac{\partial W_{di}}{\partial \delta} = \frac{1}{2n\delta^{\frac{1}{2}}}\left[\frac{\partial W_{dc}}{\partial e} - \frac{\partial W_{di}}{\partial e}\right] - \frac{n\tau\big(e - r(\tau)\big)}{3\delta^2}\left[2 + \frac{7\tau\big(e - r(\tau)\big)}{6}\right] < 0$$

where the inequality follows from proposition 7. Q.E.D.

The intuition behind proposition 7 and its corollary is as follows. An increase of e reduces the welfare level of a full abatement policy by the increase of the abatement costs, whereas at an interior solution abatement is slightly reduced, thus mitigating the adverse balance of payments effect. For large levels of e marginal costs are increasing sharply, eventually rendering full abatement inferior to a partial abatement policy. A similar argument holds for changes of δ.

Finally we turn to a comparison between abatement and domestic welfare in autarky and under free trade when $1 \leq \delta \leq 4$ and $\delta^{1/2}/n < e \leq 4/n$. We have already seen that in this case the optimal abatement level is smaller under free trade than in autarky. As far as welfare levels are concerned we get the following result.

Proposition 8: Let when $1 \leq \delta \leq 4$ and $\delta^{1/2}/n < e \leq 4/n$. For each δ there exists $\hat{e}(\delta)$ such that

$W_{di} > W_a$ for $e < \hat{e}(\delta)$ and $W_{di} \leq W_a$ for $e \geq \hat{e}(\delta)$.

Proof: We have already shown that $W_{di} > W_a$ for $e = 1/n$ and $W_{di} \leq W_a$ for $e = 4/n$ with equality for $\delta = 4$. Note further $\partial W_a/\partial e = -2 > \partial W_{di}/\partial e$ (see proof of proposition 7). Consequently, there must exist $\hat{e}(\delta)$ at which W_{di} and W_a intersect. Q.E.D.

Before turning to end–of–pipe abatement, it seems appropriate to take stock of the results obtained so far. Free trade is superior to autarky, regardless of its environmental implications, if δ and thus the gains from trade are sufficiently large. Complete abatement is optimal if δ is sufficiently large and the non-negativity constraint on profits is not violated, implying that per firm and total emissions may be smaller in autarky than in the free trade regime. If the non-negativity constraint is binding, abatement falls below the autarky level, and for δ smaller than the above mentioned critical value domestic welfare may be reduced by a move from autarky to free trade.

5. End–of–Pipe Abatement

5.1 Autarky

In this section it is assumed that emissions per unit of output can be reduced by increasing the marginal costs of production. In order to avoid confusion with the previous section we denote the reduction per unit of output achieved with an end of pipe technology as g. The costs of reducing emissions per unit of output are again assumed to be quadratic and equal to $g^2/2$. For the emission tax rate τ marginal costs of production are given by $g^2/2 + \tau(e - g)$. Since each firm will minimize its marginal costs of production we get the incentive function

$$g(\tau) = \tau \qquad (48)$$

which holds for autarky as well as for the free trade regime. In autarky the social planner wants to minimize total costs of pollution which equal $2e - g + g^2/2$, and obviously the optimal level of abatement is given by

$$g_a{*} = \min\{e, 1\} \qquad (49)$$

implying the maximum welfare level in autarky denoted as W_a

$$W_a = \begin{cases} \beta - e - \dfrac{e^2}{2} - \dfrac{\delta}{4n} \; for \; e < 1 \\ \beta - 2e + \dfrac{1}{2} - \dfrac{\delta}{4n} \; for \; e \geq 1 \end{cases} \qquad (50)$$

5.2 Free Trade

Using (48) the marginal costs of a domestic firm equal $\tau(e - \tau/2)$. Domestic and foreign equilibrium prices for a given emission tax rate are – analogously to (21) and (22) –

$$p_h = \frac{\delta}{2n} + \frac{2\tau\left(e - \frac{\tau}{2}\right)}{3}, \tag{51}$$

$$p_f = \frac{\delta}{2n} + \frac{\tau\left(e - \frac{\tau}{2}\right)}{3}. \tag{52}$$

Total domestic production equals

$$x_h = 1 - \frac{2n\tau\left(e - \frac{\tau}{2}\right)}{3\delta} \tag{53}$$

There are two crucial differences between clean technology and end–of–pipe abatement with respect to the impact of an emission tax on abatement, output and prices in a free trade regime. With end–of–pipe abatement, the incentive to reduce emissions is the same in autarky as under free trade, and it is independent of the number of firms, of the emissions per unit of output as well as of the strength of the preference for the ideal variety. Secondly, any unilateral emission tax reduces domestic production, as it increases the domestic marginal costs of production, even if there is complete abatement.

By analogy to (32), (34) and (38) we get for the components of the domestic welfare function denoted as *D*, *B* and *E*

$$D_h(\tau) = \frac{\delta}{8n} + \frac{n\tau^2}{18\delta}\left(e - \frac{\tau}{2}\right)^2 \tag{54}$$

Recall that $\delta/8n$ is the total disutility if all $2n$ firms charge the same price and the second expression on the right hand side of (54) captures the effect of higher marginal costs in the domestic country on relative prices. Differentiating (54) with respect to τ yields

$$\frac{\partial D_h}{\partial \tau} = -\frac{2n\tau\left(e - \frac{\tau}{2}\right)}{18\delta}(e - \tau) \leq 0 \tag{55}$$

The strict inequality holds for $e > \tau > 0$. The marginal effect of τ on D_h equals zero for $\tau = 0$. Total emission costs including the costs of abatement equal

$$E_h(\tau) = 2e - \tau\left(1 - \frac{\tau}{2}\right)\left(1 - \frac{2n\tau\left(e - \frac{\tau}{2}\right)}{3\delta}\right) \tag{56}$$

The second expression in brackets equals the quantity produced in the home country which is multiplied by the net reduction of pollution. Note that for $\tau \geq 2$ total emission costs are greater than without any tax and abatement at all. The first derivative equals

$$\frac{\partial E_h}{\partial \tau} = -\left\{1 - \tau - \frac{2n\tau}{3\delta}\left[\left(e - \frac{\tau}{2}\right)(1 - \tau) + (e - \tau)\left(1 - \frac{\tau}{2}\right)\right]\right\} \tag{57}$$

Note that for $\tau = 0$ this expression is equal to one. The balance of trade is given by

$$B_h(\tau) = -\frac{n\tau\left(e - \frac{\tau}{2}\right)}{3\delta}\left(\frac{\delta}{2n} + \tau\left(e - \frac{\tau}{2}\right)\right) \tag{58}$$

With the first derivative

$$\frac{\partial B_h}{\partial \tau} = -\frac{2n\tau\left(e - \frac{\tau}{2}\right)(e - \tau)}{3\delta} - \frac{e - \tau}{6} \leq 0 \tag{59}$$

Note that for $\tau = 0$ this expression equals $-e/6$. As in (55) for $e > \tau$ the strict inequality holds.

5.3 Welfare Comparisons

From (57) and (59) follows immediately

Proposition 9: In the free trade regime a welfare increasing emission tax exists for the domestic country if and only if $e < 6$.

Proof: According to (57) and (59) $W_h'(0) = -E_h'(0) + B_h'(0) = 1 - e/6$. Q.E.D.

Comparing this to Proposition 2 reveals that the range of the marginal emission rate for which one–sided emission taxes are beneficial is much wider for end–of –pipe abatement than with clean technology investments (where it is $4/n$). In particular, it is independent of the number of firms. The reason is that in the latter case fixed costs have to be incurred by each firm which are independent of its output, hence the total costs of abatement are increasing in the number of firms, whereas with end of pipe abatement only variable costs are affected, hence abatement costs for a given total output are independent of the number of firms. For the same reason, however, the condition on δ which ensures a higher welfare level with free trade than in autarky are more demanding, as the following result shows.

Proposition 10: A sufficient condition for a higher attainable domestic welfare level with free trade than in autarky is $\delta > 4n$.

Proof: Without emission tax domestic welfare with free trade equals $W_h(0) = \beta - 2e - \delta/8n$. Subtracting from this W_a as given in (50) yields $\delta/8n - \min\{e^2/2, 1/2\}$ which is positive for $\delta > 4/n$. Q.E.D.

It should be noted, however, that for $e \leq 6$ a strictly positive τ is optimal and a smaller value of δ suffices to make free trade welfare improving in comparison to autarky. We show next that with end of pipe abatement total emissions are always greater with free trade than under autarky.

Proposition 11: The tax rate τ and thus emission reduction per unit of output r is smaller than 1 for $e > 1$. Total abatement in the domestic country is smaller under free trade than in autarky.

Proof: According to Proposition 9, $W_h'(0) > 0$ for $e > 6$. Using (55), (57) and (59) we get

$W_h'(\tau) < 0$ if both, τ and e are greater than 1.

The second statement of the proposition follows from the fact that total domestic production is smaller under free trade than in autarky for $\tau > 0$ because of an increase of marginal costs.

Q.E.D.

A complication arises for $e \leq 1$. In autarky in this case total abatement is optimal. In the free trade regime full abatement requires that it is feasible, meaning that δ must be sufficiently large to ensure $x_h > 0$. Feasibility, however, is not sufficient for $\tau = e$ to be the optimal policy as for small δ the domestic welfare function may reach its maximum at some $\tau < e$. We take up both conditions on δ in turn.

As far as feasibility is concerned it can be seen immediately from (53) that $x_h > 0$ for $\tau = e$ requires $\delta > ne^2/3$. We show next that even if this inequality holds the optimal emission tax rate may be smaller than e.

Proposition 12: Suppose $e \leq 1$. The welfare maximizing tax rate is smaller than e if $\delta \leq \delta_k$ which satisfies

$$\delta_k = \frac{ne^2\left(24 - \frac{e}{2}\right)}{72 - 42e} \qquad (60)$$

Proof: It can be checked that for δ_k we get $W_h(0) = W_h(e)$. Since $W_h'(0) > 0$ there exist emission tax rates satisfying $0 < \tau < e$ such that $W_h(\tau) > W_h(e)$. Furthermore

$$\frac{\partial W_h(0)}{\partial \delta} = -\frac{1}{8n} < -\frac{1}{8n} + \frac{ne^3(24 - 5e)}{72\delta^2} = \frac{\partial W_h(e)}{\partial \delta}, \qquad (61)$$

and obviously full abatement cannot be optimal for $\delta < \delta_k$ as reducing δ has a stronger positive effect on $W_h(0)$ than on $W_h(e)$. Q.E.D.

Note that $\delta_k > ne^2/3$ for $0 < e \leq 1$. We show next that for each $e \leq 1$ some value of δ exists such that the optimal tax rate equals e.

Proposition 13: Suppose $0 < e \leq 1$. There exists some finite $\delta > \delta_k$ such that the optimal emission tax rate equals e.

Proof: Denote an interior maximum of $W_h(\tau)$ as W_{di}. We know from Proposition 12 that for δ_k $W_{di} > W_h(0)$. By the envelope theorem we get

$$\frac{\partial W_{di}}{\partial \delta} = -\frac{1}{8n} + \frac{n\tau^2}{72\delta^2}(48e - 24\tau - 52\tau e + 19\tau^2 + 28e^2) \quad (62)$$

Note that for $\tau = e$ this is equal to $\partial W_h(0)/\partial\delta$ as shown in (60). We show next that $\partial W_h(0)/\partial\delta > \partial W_{di}/\partial\delta$ for $\tau < \delta$. To see this differentiate (62) with respect to τ:

$$\frac{\partial^2 W_{di}}{\partial\delta\partial\tau} = \frac{4n\tau}{72\delta^2}(14e^2 + 24e - 18e\tau - 18\tau + 19\tau^2) > 0 \quad (63)$$

The inequality follows from

$24e - 18\tau \geq 6e$,

$e[18\tau - 14e] < 4e$.

As W_{di} decreases in δ much faster than $W_h(0)$ the two must intersect at some value of

$\delta > \delta_k$. Q.E.D.

Figure 6 illustrates the optimal tax rate for small δ.

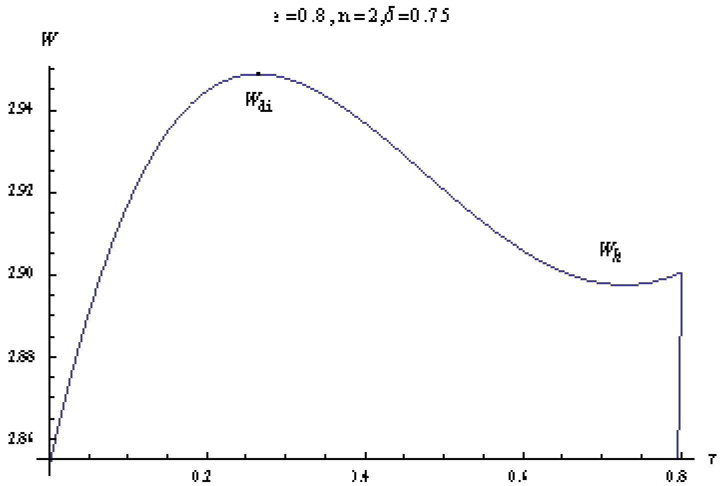

Figure 6

We turn next to the case with δ sufficiently large to render a corner solution, i.e. full abatement and tax rate $\tau = e$. Note that a tax rate equal to one would also lead to efficient abatement for $e < 1$, hence we get the standard result that the marginal emission tax rate is equal to the marginal damage caused by emissions. The downside is, of course, that even with an efficient emission tax, total abatement is reduced by a move from autarky to free trade, and this environmental deterioration may outweigh the gains from trade even for small levels of e. This is illustrated in Figure 7 which depicts $W_h(\tau)$ and W_a for $\delta = 2$, $n = 2$ and $e = 0.8$. Abatement is complete and efficient at the firm level, but total emissions are greater than in autarky, and together with the negative balance of trade effect this leads to a lower welfare level in the free trade regime than under autarky.

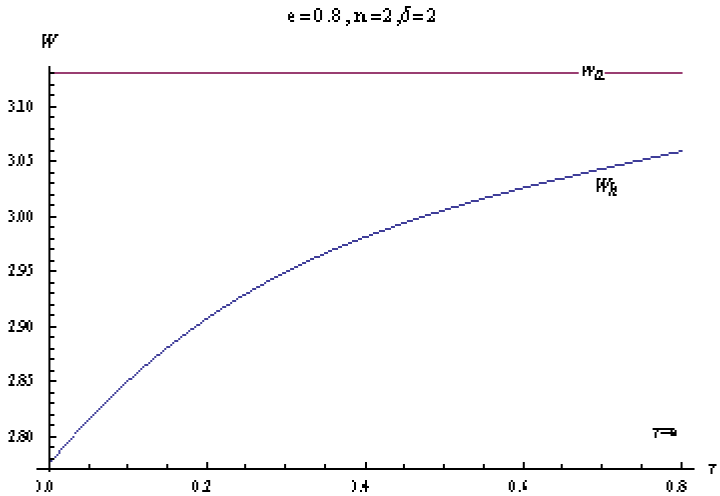

Figure 7

It is noteworthy that for large values of δ no interior local maximum exists. For larger δ a higher welfare level is attainable with free trade than in autarky. This is illustrated in Figure 8 with $\delta = 8$ and e = 1.5. Note that $\delta < 8n$, the level which is sufficient for gains from trade.

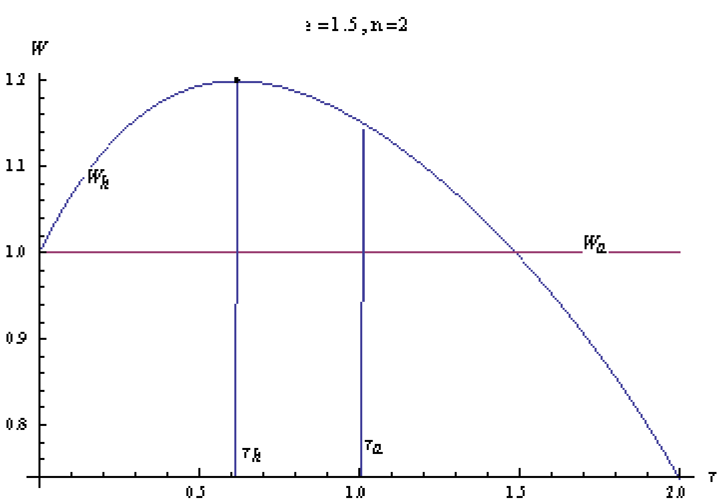

Figure 8

To sum up the results of this section, we note that an end–of–pipe abatement technology is quite likely to make a (small) emission tax welfare improving in the free trade regime, but total emissions are always greater than in autarky, and for trade to be welfare increasing, the gains due to a larger number of varieties have to be substantial relative to emissions per unit of output.

6. Summary and Conclusions

We have shown that in a model of intra-industry trade based on the ”ideal variety” approach, the effect of one–sided emission taxes on emissions and welfare in that country are ambiguous and depend crucially on the available abatement technology. If marginal costs of production are not affected, as has been assumed in the clean technology approach, then abatement may even be larger in the free trade regime than in autarky. On the other hand, with end–of–pipe abatement, emission taxes look more likely to be beneficial in the free trade regime, but total abatement is smaller than in autarky, and free trade looks less likely to be better than autarky in terms of domestic welfare.

As far as policy conclusions are concerned, purely theoretical models like the ones in this paper have to be considered with care. They are useful for highlighting some aspects, but they neglect others. Some of those may be addressed within the present framework, but others obviously require different approaches. We shall discuss some of these issues in turn.

One restriction of the foregoing analysis is the focus on emission taxes as the only instrument of environmental policies. Clearly, several others are available and have been used (see Aldy and Stavins (2011) for a recent survey). In principle, they can be used in the present framework, and this should be the subject of further research.

Another rather strong assumption concerns the fixed number of firms and the ruling out of relocations of firms between countries. Both aspects are difficult to tackle in the present model for similar reasons. If the number of firms changes in the two countries after moving from autarky to free trade, the location of firms, both in the two countries and in the space of product varieties, becomes a very intricate issue with no obvious solution. It is hard to tell how important this aspect really is as exit costs, relocation costs and repercussions on the factor markets have to be taken into account. Clearly if not only output per domestic firm but also the number of domestic firms is reduced the effectiveness of unilateral environmental policies in one country is further reduced.

While countries may differ with respect to their willingness to promote environmental protection, it is unlikely that one country remains completely passive. Strategic interactions between governments, which have been analyzed in earlier papers, could be accommodated in the present framework.

In any case, what seems to be certain is that an efficient solution of a global problem like the emission of uniformly mixing pollutants requires international co-operation. Whether in its absence individual countries should still implement their own policy measures, or whether even free trade should be restricted, remains an open question, but some comfort may be derived from the fact that the foregoing analysis has shown that beneficial one-sided environmental policies are available under a wide variety of circumstances, without sacrificing potential gains from trade.

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