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The Harvard Environmental Economics Program

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Technology Choice and Capacity Portfolios under Emissions Regulation

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We study the impact of emissions tax and emissions cap-and-trade regulation on a firm’s long-run technology choice and capacity decisions. We study the problem through a two-stage, stochastic model where the firm chooses capacities in two technologies in stage one, demand uncertainty resolves between stages (as does emissions price uncertainty under cap-and-trade), and then the firm chooses production quantities. As such, we bridge the discrete choice capacity literature in Operations Management (OM) with the emissions-related sustainability literature in OM and Economics. Among our results, we show that a firm’s expected profits are greater under cap-and-trade than under an emissions tax due to the option value embedded in the firm’s production decision, which contradicts popular arguments that the greater uncertainty under cap-and-trade will erode value. We also show that improvements to the emissions intensity of the “dirty” type can increase the emissions intensity of the firm’s optimal capacity portfolio. Through a numerical experiment grounded in the cement industry, we find emissions to be less under cap-and-trade, with technology choice driving the vast majority of the difference.

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1. Introduction

Cap-and-trade emissions regulation was implemented within Europe in 2005 under the European Union Emissions Trading Scheme (EU-ETS). Similar legislation has been passed regionally within the US in both California and the Northeast. Meanwhile, a growing chorus of voices that include politicians and economists (Inglis and Laffer 2008), business leaders (Pontin 2010), and climate skeptics (Jowit 2010) have called for an emissions tax regime rather than cap-and-trade. As a result, several bills within the US Congress propose national emissions regulation. Some of these recommend a cap-and-trade mechanism (e.g., H.R 1666, H.R 1759 and H.R 2454), while others recommend an emissions tax (e.g., H.R 594, H.R 1337, and H.R 2380).

With this activity on the policy front, firms wrestle with how to manage in emissions regulated environments. Given the capital intensity of emissions regulated sectors, of principle interest are long-term capital decisions such as the technologies that firms choose when building capacity. Such decisions determine how firms trade off traditional operating and investment costs with emissions intensity (i.e., the emissions generated per unit of production). A firm’s emissions intensity determines its exposure to uncertain allowance prices if they are regulated by a cap-and-trade regime,
or a constant unit emissions cost under an emissions tax. Under the forthcoming Phase III of the
EU-ETS, cement executives that we met with expected production cost increases of over 40% due
to projected allowance pricing, an increase sufficient to turn profits negative at current pricing.

Given that firms may choose to operate a single technology or multiple technologies under
emissions regulation, capacity portfolios are an important aspect of this problem, but an aspect
that has largely not been considered within the existing literature. We study that question and its
derivatives here: How should a firm incorporate “clean” and “dirty” technologies into its long-term
capacity portfolio, and what are the implications for firm profits and the regulation’s effectiveness
in incentivizing clean technology adoption and emissions abatement?. We address this question
through a single-period, two-stage model of a profit-maximizing firm. In the cap-and-trade setting,
the firm develops its capacity portfolio by choosing a mix of two technologies while facing uncertain
demand and an uncertain emissions price. In the emissions tax setting, they choose the capacities
of the two technologies while facing uncertain demand and a constant tax per unit of emissions.

2. Relation to the Literature

Technology choice, capacity investment under uncertainty, the potential for multiple technologies
to be included in the optimal capacity portfolio, and the emissions context are all central to the
problem studied here – the discrete technology profile of a firm’s capacity portfolio under emissions
regulation, and the ensuing profit and emissions implications. By incorporating these elements, we
contribute to the literature by modeling that problem, and through results related to:

1. **Expected profit comparison**: Proponents of an emissions tax versus cap-and-trade regu-
lation argue that the uncertainty in emissions price under the latter regime will erode firm profits.
We show the opposite to be true. Assuming that the emissions tax is no less than the expected
emissions price under cap-and-trade, expected profits will be greater under cap-and-trade. This
results from the firm’s ability to choose not to produce under emissions price outcomes that would
be unprofitable. By recognizing the real option embedded in the capacity and production decision,
the uncertainty in emissions price under cap-and-trade creates value for the firm, rather than erodes
value. This result is driven by the uncertainty in emissions price under cap-and-trade, which is an
aspect of the practical problem that has thus far not been addressed within the literature.

2. **Investment and production subsidy**: The extant literature exploring the effect of clean
technology subsidies does so primarily in a deterministic setting, with an aggregated marginal cost
rather than an investment and production cost. We differentiate between the two costs and show
that the firm’s capacity decision is more sensitive to an investment subsidy than it is to the same
magnitude production subsidy. We also identify the conditions under which the emissions potential
of capacity increases as a result of subsidies for both clean technology investment and production.
Differences in investment and production subsidies are driven by the uncertainty faced by the firm at the time of capacity investment, which is absent in much of the emissions regulation literature.

3. **Optimal technology share:** Our research also points to a number of nuances or potential unintended consequences related to technology share. These contributions include conditions when an increase in tax rate will decrease investment in clean technology, and conditions when reducing a dirty technology’s emissions intensity can have the adverse effect of increasing the emissions intensity of the log-term optimal capacity portfolio. For these share-driven results to be non-trivial, it must be possible for multiple technologies to be included in the firm’s optimal portfolio, which has not traditionally been the case within the literature exploring emissions regulation effects.

In addition to the contributions above, we conduct a numerical experiment grounded by field research in the cement industry. Among the results, we find that emissions are 11.7% less under cap-and-trade than an emissions tax given the choice between traditional and carbon capture and storage kilns, with 85% of this effect resulting from technology choice and the remainder accounted for by a decrease in production. This analysis also illustrates that a modest emissions reduction – a 2.2% reduction in emissions under a tax, and a 12.5% reduction under cap-and-trade – can come at a steep cost for a given sector, with profits reduced by nearly 50% under both regimes.

Within the general Operations Management (OM) literature, the newsvendor networks stream (e.g., Harrison and Van Mieghem 1999, and Van Mieghem and Rudi 2002) provides a general model of technology choice and capacity investment that accommodates multiple resources and multiple products, with each product facing uncertain demand. Van Mieghem (2003) provides a review of the newsvendor network and other OM capacity literature. This framework has been used to explore a broad set of questions ranging from the relationship between financial hedging and operational flexibility (Chod et al. 2010), to technology and capacity decisions in a multi-product setting with correlated demand (Goyal and Netessine 2011), to the effect of imperfect capital markets on technology choice and capacity decisions (Boyabatli and Toktay 2011), among others. Our work is related to this literature, as is evident by the newsvendor structure to solutions in the tax setting. In other structurally-related OM work, Wang et al. (2011) study capacity portfolios under demand and cost uncertainty when investment cost is negligible, which does not pertain here as emissions-regulated sectors are capital-intensive (e.g., power generation, cement, steel, pulp-and-paper, etc.) and capacity investments must be made under uncertainty, greatly increasing the complexity of the managerial problem faced by firms in such settings. In general, the OM technology choice and capacity literature has not explored the effects of emissions regulation, and therefore it neither parameterizes emissions intensity nor addresses the focal questions explored here.

In the Sustainable OM literature related to emissions regulation without technology choice, Subramanian et al. (2007) study abatement investment in a deterministic oligopoly with an endogenous
emissions price but with capacity treated implicitly (and no capacity investment). Benjaafar et al. (2010) study the integration of emissions into procurement, production and inventory decisions in a deterministic setting without capacity investment. Keskin and Plambeck (2011) study the allocation of emissions from processes to co-products. Cachon (2011) studies retailer location and sizing to minimize transport costs inclusive of emissions social costs. Chen and Benjaafar (2012) study emissions from a two-echelon supply chain, finding that taxing each firm can lead to an increase in emissions. Within the Sustainable OM literature that explicitly considers technology choice under emissions regulation, Krass et al. (2010) model a Stackelberg game where a regulator sets a tax rate and a firm selects production technology and price. Zhao et al. (2010) explore the impact of allowance allocation schemes in a cap-and-trade setting on equilibrium production under perfect competition. Aflaki and Netessine (2011) explore the effect of intermittent supply on the adoption of renewable energy. Drake (2011) studies the effect of asymmetric emissions regulation and border adjustment in a model of imperfect competition. Caro et al. (2011) study joint abatement efforts by supply chain partners (abstracting from production and capacity decisions), finding that emissions must be over-allocated among firms in order to induce optimal abatement effort from each. In Krass et al. (2010), Zhao et al. (2010) and Drake (2011) each firm selects a single technology in deterministic settings. The same is implicitly true in Caro et al. (2011) at the firm-process level.

Within Economics, the analytic foundation for the discrete technology choice problem with demand uncertainty was laid in Crew and Kleindorfer (1976) within the peak-load pricing literature, which is reviewed in Crew et al. (1993). This framework considers the development of capacity to serve stochastic demand from a finite portfolio of technologies with heterogeneous investment and production costs. As such, the stream is structurally similar to our carbon tax setting, except production costs in our setting include both direct costs as well as unit emissions costs. The latter are determined by the emissions intensity of the technology and the tax rate. Explicitly parameterizing emissions intensity and tax rate allows us to study environmental and regulatory effects under a carbon tax.

Within Environmental Economics, the technology-related research (see Jaffe et al. 2003 for a review) primarily treats the problem in a deterministic setting, abstracting away from the uncertainty that firms face when choosing technologies and investing in capacity, with firms selecting a single technology (i.e., cost curve) as a consequence. Carraro and Soubeyran (1996) provide an exception with the firm conditionally operating a mixed portfolio, but capacity is treated as exogenous, and an upper bound is imposed on clean capacity. Requate (1998) explores technology choice under emissions regulation in a competitive setting, but also does so deterministically with an innovating firm selecting a technology from a continuum of options while competing firms each operate a common legacy technology. Requate and Unold (2003), and Requate (2005) also treat the
problem deterministically in a competitive setting where each firm selects a single technology. Zhao (2003) and Krysiak (2008) treat technology choice under emissions regulation in a competitive and dynamic setting. In both of these papers, each firm selects a single technology from a continuum of infinite options rather than selecting a portfolio from among discrete choices. Further, Requate (1998), Requate and Unold (2003), and Zhao (2003) do not include capacity and output as firm decisions, and Requate (2005) abstracts away from capacity investment costs.

Our work methodologically relates to the OM capacity literature, within which technology choice and capacity investment under uncertainty play central roles, but which does not consider emissions regulation. Our work also contextually relates to the Sustainability literature in OM and Economics, which both include streams focused on emissions regulation, but abstract from investment under uncertainty and the potential for firms to include multiple technologies in their capacity portfolio. By bridging these bodies of work, we develop novel insights relevant to both policy and practice.

3. Technology Choice under Emissions Tax Regulation

We model emissions regulation in a setting where the firm commits to a capacity portfolio under demand uncertainty, but produces after this uncertainty has resolved. This reflects the serve-to-order environment faced by power generators (responsible for 70% of Europe’s regulated emissions), and the environment in capital-intensive, mature industries where the primary drivers of uncertainty are economic and fairly well-known in the short-run. The latter applies to emissions regulated sectors such as cement, steel, glass, and pulp-and-paper. Structurally, this results in a newsvendor framework, which aligns with other OM capacity investment literature (e.g., Harrison and Van Mieghem 1999, Cachon and Lariviere 1999, Chod and Rudi 2005, Boyabatli and Toktay 2011, and Goyal and Netessine 2011) as well as capacity investment literature in economics (e.g., Crew and Kleindorfer 1976, Chao 1983, and Kleindorfer and Fernando 1993).

The emissions tax model is a special case of the cap-and-trade model discussed in Section 4. However, it is an important case worthy of close inspection. We explore that setting here for its tractability and practical relevance – three bills before the 111th US Congress proposed a carbon tax (H.R 594, H.R 1337, and H.R 2380). Under a tax, emission-regulated firms would be charged a constant tax rate for each unit of emissions that they generate. We also address clean technology subsidies – effectively a negative tax for clean investment and/or production – within Section 3.2.

3.1. Emissions tax model

To address the technology choice problem under such regulation, we employ a two-stage, single period model. During the first stage, a risk-neutral firm builds its capacity portfolio from two technology types, with each type defining marginal investment cost, operating cost, and emissions intensity parameters, \( \beta_i \geq 0, b_i \geq 0 \), and \( \alpha_i \geq 0 \), respectively, where \( i \in N = \{1, \ldots, n\} \), with \( n = 2 \)
in our case. Without loss of generality, we assume type 1 is the “dirty” technology and type 2 is the “clean” technology – i.e., \( \alpha_1 \geq \alpha_2 \).

In stage one, the firm makes capacity decisions, \( K_i \), to maximize expected profit, incurring investment costs, \( K_i \beta_i \). Between stage one and stage two, stochastic demand, \( \tilde{D} \), is realized. Within the second stage, the firm serves realized demand, \( d \), by choosing non-negative production quantities, \( q_i \leq K_i \), to maximize operating margin. Underlying this sequence of events is the observation that the demand uncertainty faced by the firm at the time of their capacity investment decision would largely have resolved by the time of production. The firm collects price, \( p \), for every unit sold, incurs a rationing penalty, \( r \), for every unit of unmet demand, and incurs unit production costs, \( b_i + \alpha_i \tau \), where \( \tau \geq 0 \) is the emissions tax rate. Therefore the firm earns unit operating margin \( \eta_i(\tau) = p + r - b_i - \alpha_i \tau \) for each unit produced with technology \( i \). For clarity, note that we refer to operating costs, \( b_i \), emissions costs, \( \alpha_i \tau \), and production costs, \( b_i + \alpha_i \tau \).

Most industries currently under emissions regulation are price-regulated and/or highly commoditized (e.g., power generation, glass, steel, cement, ceramics, pulp and paper, etc.), so we assume above that the firm is a price-taker\(^1\). To avoid trivial outcomes, we assume \( \eta_i(\tau) - \beta_i > 0 \), \( \forall i \) and \( \beta_1 > \beta_2 \). The former condition ensures that each type is profitable, and the latter condition ensures that type [2] is not trivially dominated by type [1] (i.e., if the type preferred in merit order, [1], is also the lower investment cost technology, then type [2] could be excluded from the optimal portfolio without further analysis). All proofs are provided in Appendix 1.

**Second stage problem.** In the second stage, the firm maximizes operating margin

\[
\max_{q_1, q_2} \pi(K, d, \tau) = \max_{q_1, q_2} \left( \sum_{i=1}^{2} \eta_i(\tau)q_i \right) - rd
\]

s.t. \( 0 \leq q_i \leq K_i \), \( \forall i \)

\[
\sum_{i=1}^{2} q_i \leq d.
\]

Let \( \xi(\tau, i) \) order technology types given emissions price realization, \( e \), with ties broken in favor of technology 1, so that

\[
\xi(\tau, 1) = \min\{\arg\min_{i \in N} b_i + \alpha_i \tau\} \quad \text{and} \quad \xi(\tau, 2) = N \setminus \xi(\tau, 1).
\]

\(^1\) Much of the OM technology choice literature treats price exogenously as we do (e.g., Harrison and Van Mieghem 1999, Netessine et al. 2002, and Tomlin and Wang 2005). In order to explicitly characterize capacities, the discrete technology choice literature with stochastic demand that does endogenize price either overtly assumes that firms utilize all of their capacity (i.e., Goyal and Netessine 2007), or assumes that the firm is a monopoly (i.e., Bish and Wang 2004, Goyal and Netessine 2011, and Boyabatli and Toktay 2011) in which case production and capacity equivalency is a result. In the emissions regulated setting that we explore, these assumptions would be specious. Emissions regulated sectors are highly commoditized with firms reflecting price-takers more than monopolists, and emissions price uncertainty can resolve so that it would be unprofitable for a firm to utilize a given technology.
Let $K(\tau)$ represent the vector of capacities for a given $\tau$, ordered in terms of increasing production costs through $\xi(\tau, i)$, i.e., $K(\tau) = (K_1(\tau), K_2(\tau))$, where $b_\xi(\tau, i) + \alpha_{\xi(\tau, i)} \tau \leq b_\xi(\tau, 2) + \alpha_{\xi(\tau, 2)} \tau$. For brevity, we will use the notation $[i]$ for $\xi(\tau, i)$, but the reader should note that merit ordering depends on the realized emissions price. With constant tax rate, $\tau$, there is a single possible merit order within the tax setting. Therefore, where $(a)^+ = \max(0, a)$, the firm maximizes operating margin by choosing quantities

$$q_{[i]} = \min \left( K_{[i]}, \left( d - \sum_{k=1}^{i-1} K_{[k]} \right)^+ \right). \quad (2)$$

First stage problem. In the first stage of the emissions tax setting, with $\tau$ known ex-ante, the firm faces a single uncertainty, demand $\tilde{D}$. Therefore, the firm maximizes expected profits by solving

$$\max_{K_1, K_2} \Pi(D, \tau) = \max_{K_1, K_2} \mathbb{E} \left[ \pi \left( K, \tilde{D}, \tau \right) \right] - \sum_{i=1}^{2} \beta_i K_i \quad \text{s.t. } K_i \geq 0, \forall i. \quad (3)$$

Concavity of $\Pi$ is proven in Appendix 1. As the cross-partial is negative, the upper bound of $K_{[i]}$ in the emissions tax setting, $\bar{K}_i^{T}$, is determined when $K_{[-i]} = 0$, so $\bar{K}_i^{T} = F^{-1}_\tilde{D} \left( 1 - \frac{\beta_{[i]}}{\eta_{[i]}(\tau)} \right)$.

Solution. Letting $F_\tilde{D}$ represent the CDF of the demand distribution, the following proposition summarizes the solution to the emissions tax capacity decision;

**Proposition 1.** Under emissions tax regulation where type $i$ dominates merit order, the optimal capacities, $K^*_i$ and $K^*_{-i}$, are characterized by

$$K^*_i = \begin{cases} 
\bar{K}_i^{T} & \text{if } \beta_{-i} \geq \frac{\eta_i(\tau)}{\eta_{-i}(\tau)} \\
0 & \text{if } \eta_i(\tau) - \beta_i \leq \eta_{-i}(\tau) - \beta_{-i} \\
F^{-1}_\tilde{D} \left( 1 - \frac{\beta_i - \beta_{-i}}{\eta_i(\tau) - \eta_{-i}(\tau)} \right) & \text{otherwise},
\end{cases}$$

and

$$K^*_{-i} = \begin{cases} 
\bar{K}_{-i}^{T} & \text{if } \beta_{-i} \geq \frac{\eta_i(\tau)}{\eta_{-i}(\tau)} \\
0 & \text{if } \eta_i(\tau) - \beta_i \leq \eta_{-i}(\tau) - \beta_{-i} \\
F^{-1}_\tilde{D} \left( 1 - \frac{\beta_{-i}}{\eta_{-i}(\tau)} \right) - K^*_i & \text{otherwise}.
\end{cases}$$

Observe that type $[1]$ is excluded from the optimal portfolio despite its production cost advantage if profit margin increases in merit order. Under this condition, the operating margin advantage type $[1]$ has over type $[2]$ is dominated by its investment cost disadvantage. Type $[2]$ would generate more profit per unit of capacity, and require less investment. Conversely, type $[2]$ technology will be excluded from the optimal portfolio under an emissions tax if its investment cost is not a sufficiently small fraction of type $[1]$ investment cost – because it earns lower operating margin, it must present less ex-ante risk to warrant inclusion in the firm’s optimal portfolio. It is also
clear from Proposition 1 that, for interior solutions, the firm invests in total capacity equivalent to the newsvendor solution of its least preferred type, i.e., the type that it would utilize last, given sufficiently high demand.

3.2. Key results within an emissions tax setting

Observe that \( K_i \) decreases monotonically in its own investment and operating costs \( \beta_i \) and \( b_i \), respectively. Further, for interior solutions, \( K_i \) increases monotonically in type \(-i\) investment and operating costs, \( \beta_{-i} \) and \( b_{-i} \). Symmetric results hold for capacity \( K_{-i} \). While these results are not surprising, they have relevance with respect to clean technology subsidies. A social planner seeking to catalyze investment in clean technology would have interest in knowing the relative impact of subsidizing clean investment versus clean production. We explore relative differences given a per unit subsidy, \( \delta > 0 \), applied to per unit investment cost \( \beta_2 \) and production cost \( b_2 \).

**Corollary 1.** Assume a per unit subsidy, \( \delta \), without a binding budget constraint. Then, regardless of merit order,

- a) \( K_1^* \) decreases in \( \delta \) at a greater rate under investment subsidy than production subsidy.
- b) \( K_2^* \) increases in \( \delta \) at a greater rate under investment subsidy than production subsidy.

And when type 1 is preferred in merit order,

- c) \( K_1^* + K_2^* \) increases in \( \delta \) at a greater rate under investment subsidy than production subsidy.

Federally, the US subsidizes both clean technology investment (e.g., the Business Energy Investment Tax Credit) and production (e.g., the Renewable Electricity Production Tax Credit). Most US states also offer their own clean investment and production subsidies (e.g., there are over 140 US state-level corporate tax and rebate incentives for renewable energy alone). Many of these subsidies do not include a budget constraint (e.g., state-level renewable energy sales tax exemptions), or include a budget constraint that is non-binding. Corollary 1 addresses these settings.

By Corollary 1a and b, investment subsidies are a more sensitive mechanism than production subsidies for a policy-maker interested in decreasing investment in dirty technology, or increasing investment in clean technology. The intuition behind the result is that the firm will benefit from an investment subsidy for each unit of subsidized capacity that they add to their portfolio, but will only benefit from a production subsidy for those units that they utilize. Thereby, the firm is also motivated to install less dirty technology (the un-subsidized type) under an investment subsidy than under the same magnitude production subsidy. The firm is also motivated to add more clean technology under an investment subsidy than under the same magnitude production subsidy. The latter result (Corollary 1b) also implies that the total subsidy issued would be greater under an investment subsidy than under a production subsidy.
Investment and production subsidies also differ in their impact on total capacity, with total capacity again increasing at a greater rate under an investment subsidy than under a production subsidy if the dirty technology, type 1, is preferred in merit order. When type 2 is preferred in merit order, total capacity is unchanged in per unit subsidy $\delta$ until it is sufficiently large to force a boundary condition (i.e., $K_1^* = 0$), with the condition binding at a lower per unit investment subsidy than production subsidy. With the boundary conditions binding, total capacity increases in $\delta$ at a greater rate when investment cost is subsidized than when production is subsidized.

The results of Corollary 1 are driven by the uncertainty that the firm faces at the time of capacity investment – without this uncertainty, production and investment subsidies would have identical effects on the optimal capacity portfolio. The extant literature exploring the impact of subsidies on technology choice and capacity investment does so primarily in a deterministic setting, with an aggregated marginal cost rather than an investment and production cost (e.g., Baumol and Oates 1975, Milliman and Prince 1989, and Fischer and Newell 2008). Separating marginal cost into investment and production costs, with the investment cost incurred under uncertainty, we contribute to the subsidy literature by illustrating that investment and production subsidies have a different magnitude effect, with the optimal capacity portfolio more sensitive to the former than the latter. Further, subsidies can have an adverse effect on the emissions potential of capacity, $\alpha_1 K_1 + \alpha_2 K_2$, summarized by the following corollary where $f_D$ is the probability density of demand.

**Corollary 2.** Assume $\alpha_2 > 0$, an interior solution and a per unit subsidy, $\delta$, without a binding budget constraint. If type 1 is preferred in merit order, then the emissions potential of capacity

a) Increase due to investment subsidy iff

$$\frac{\alpha_1 - \alpha_2}{\alpha_2} \leq \frac{f_D \left( 1 - \frac{\beta_1 (\eta_1 t) - \beta_2 (\eta_2 t)}{\eta_1 (t) - \eta_2 (\tau)} \right)}{f_D \left( 1 - \frac{\beta_2 (\eta_2 t)}{\eta_2 (\tau)} \right)} \eta_2 (t).$$

b) Increase due to production subsidy iff

$$\frac{\alpha_1 - \alpha_2}{\alpha_2} \leq \frac{f_D \left( 1 - \frac{\beta_1 (\eta_1 t) - \beta_2 (\eta_2 t)}{\eta_1 (t) - \eta_2 (\tau)} \right)}{f_D \left( 1 - \frac{\beta_2 (\eta_2 t)}{\eta_2 (\tau)} \right)} \frac{\beta_2}{\beta_1 - \beta_2} \left( \frac{(\eta_1 (t) - \eta_2 (\tau))^2}{\eta_2 (t)} \right).$$

Subsidization of clean technology increases the clean technology share, $\psi_2 = \frac{K_2}{K_1 + K_2}$, of the optimal capacity portfolio. This partial substitution of clean technology for dirty technology within the firm’s capacity portfolio contributes to reducing the emissions potential of capacity. However, by Corollary 1c, the subsidy also increases total capacity, which increases emissions potential. If the emissions benefits resulting from the shift in technology share dominate the emissions resulting from the increase in capacity, then emissions potential decreases. However, if the percent difference in the emissions intensity of the dirty type relative to the clean type is less than the condition stipulated in the RHS of Corollary 2a or b, then the total emissions potential of capacity increases as a result of an investment or production subsidy, respectively. Under such conditions, the emissions potential of the incremental capacity dominates the emissions benefit due to the shift in technology share. If type 2 is preferred in merit order, then the subsidy leads to an increase in the optimal
clean technology share without an increase in total capacity for all interior solutions. As a result, the emissions potential of capacity strictly decreases under such conditions. However, if type 2 dominates the optimal portfolio (i.e., if $K^*_1 = 0$), then investment or production subsidy trivially increases emissions potential.

Corollary 2 complements two “perverse effects” that can result from environmental subsidy. In a single-technology setting under perfect competition, Bramhall and Mills (1966) describe and Baumol and Oates (1975) prove that an emissions abatement subsidy reduces the emissions of a firm, but increases the emissions of an industry due to increased entry. In a competitive setting with technology choice, Fischer and Newell (2008) explain and Hutchinson et al. (2010) prove that a marginal cost subsidy applied to clean technology decreases equilibrium price. This results in an increase in consumption that can conditionally dominate emissions reductions achieved through a switch to the subsidized technology. Corollary 2 differs from, and complements, these previously identified perverse effects. The results of Corollary 2 are driven by demand uncertainty rather than pricing equilibrium or entry. In our setting, a clean technology subsidy increases the profit earned for each unit of fulfilled demand, so the firm optimally increases capacity to serve a greater fraction of the demand distribution. This, in turn, conditionally offsets emissions improvements that result from increased clean technology share.

Where clean technology investment and production subsidies increase the profitability of type 2 technology, an emissions tax decreases profitability for both types, moderated by their respective emissions intensities. As a consequence, the effect of an emissions tax on capacities is more nuanced than the effect of investment and production subsidies.

Corollary 3. Assume an interior solution under an emissions tax regime. Then

a) Optimal dirty capacity, $K^*_1$, decreases monotonically in tax rate, $\tau$.

b) Optimal clean capacity, $K^*_2$, increases monotonically in $\tau$ when it is preferred in merit order.

c) When not preferred in merit order, optimal clean capacity

$$K^*_2 \begin{cases} 
\text{decreases monotonically in } \tau \text{ if } & \frac{\alpha_1 - \alpha_2}{\alpha_2} \leq \frac{f_D(1 - \frac{\beta_1}{\eta_1(t)})}{f_D(1 - \frac{\beta_2}{\eta_2(t)})} \frac{\beta_2}{\beta_1} \frac{(\eta_1(t) - \eta_2(t))^2}{(\eta_2(t))^2} \\
\text{increases monotonically in } \tau \text{ otherwise.} 
\end{cases}$$

The conditions where $K^*_2$ decreases in $\tau$ stipulate a percent difference in emissions intensity between types. If this difference is less than the characterized threshold, then both type 1 and type 2 capacity decrease in $\tau$ (given that type 2 is not preferred in merit order). However, if the difference in emissions intensity between the two types is greater than the percent threshold, then type 2 capacity is added to compensate for decreases in type 1 capacity. The effect on type 1 capacity is clear. The added cost incurred by type 1 production due to the increase in emissions tax
results in a lower marginal profit, which in turn leads the firm to invest in less type 1 capacity. The first order effect on type 2 capacity is also clear – a decrease in type 2 capacity as a consequence of the increased production cost. However, a second order effect works in the opposite direction. The decreased investment in type 1 capacity (the preferred technology in the conditional result above) leaves a greater portion of potential demand unserved by type 1 technology, thereby increasing the marginal revenue for type 2 technology, which results in the firm investing in more type 2 capacity. When the above condition holds, the second order effect dominates and type 2 capacity increases as a consequence of the increased tax rate.

The conditions under which an increase in emissions tax leads to a decrease in clean capacity (Corollary 3c) are identical to those under which an increase in a production subsidy leads to an increase in the emissions potential of capacity (Corollary 2b). Therefore, when type 1 is preferred in merit order and the stipulated condition holds, the policy-maker cannot increase clean capacity without also increasing emissions potential (or decrease emissions potential without also decreasing clean capacity). When the stipulated conditions do not hold, the policy-maker can increase clean capacity and decrease the emissions potential of capacity with either a tax or subsidy.

Under a boundary solution, the technology that monopolizes the optimal portfolio decreases in \( \tau \). Together with Corollary 3, this indicates that \( K^*_2 \) is conditionally non-monotonic in \( \tau \). Krass et al. (2010) find a non-monotonic effect of tax rate on clean technology adoption as well, but their result is quite different from that above. The Krass et al. result pertains to the adoption of a single technology only, rather than a potential portfolio of technologies, and is a consequence of demand decreasing in response to emissions costs driving an increase in price. Corollary 3, on the other hand, follows directly from the potential for a mixed portfolio, with an interior solution (i.e., a mixed technology portfolio) as a necessary condition for \( K^*_2 \) to increase in \( \tau \).

Changes in emissions intensity for type \( i \) will have the same directional impact on optimal capacities as a change in type \( i \) operating or investment cost. However, the impact such changes can have on the emissions intensity of the firm’s portfolio, \( \nu_K = \frac{\alpha_1 K^*_1 + \alpha_2 K^*_2}{K^*_1 + K^*_2} \), can be surprising\(^2\).

**Corollary 4.** Given an interior solution under an emissions tax regime, an improvement in emissions intensity will lead to a strict increase in the emissions intensity of the firm’s capacity portfolio when a) the dirty technology is preferred in merit order (i.e., a decrease in \( \alpha_{[1]} \)) and

\[
K^*_1 < \frac{\beta_{[1]}(\alpha_{[2]} - \alpha_{[1]})}{f_D \left( 1 - \frac{\eta_{[1]}(\tau) - \eta_{[2]}(\tau)}{Q_{[1]}(\tau) - Q_{[2]}(\tau)} \right)} \left( \eta_{[1]}(\tau) - \eta_{[2]}(\tau) \right)^2,
\]

and b) when the dirty technology is not preferred in merit order (i.e., a decrease in \( \alpha_{[2]} \)) and

\[
K^*_2 < \frac{K^*_1}{K^*_1 + K^*_2} \frac{\beta_{[2]}(\alpha_{[2]} - \alpha_{[1]})}{f_D \left( 1 - \frac{\eta_{[2]}(\tau)}{Q_{[2]}(\tau)} \right)} \eta_{[2]}(\tau) + \frac{\beta_{[1]}(\alpha_{[2]} - \alpha_{[1]})}{f_D \left( 1 - \frac{\eta_{[1]}(\tau) - \eta_{[2]}(\tau)}{Q_{[1]}(\tau) - Q_{[2]}(\tau)} \right)} \left( \eta_{[1]}(\tau) - \eta_{[2]}(\tau) \right)^2.
\]

\(^2\)The expected emissions intensity of the firm’s production, \( \nu_q = \frac{\alpha_{[1]} q_{[1]} + \alpha_{[2]} q_{[2]}}{q_{[1]} + q_{[2]}}, \) is a more relevant metric. However, we use the emissions intensity of the firm’s portfolio, \( \nu_K \), for its tractability and its value as a proxy for \( \nu_q \).
The conditions in Corollary 4 describe the threshold where improvements to the emissions intensity of the dirty type lead to greater emissions intensity for the portfolio as a whole. This is surprising given that, by definition, such improvements reduce per unit emissions from the dirty type: type [1] when the dirty type is preferred, and type [2] when it is not preferred. However, there is a second order effect on technology share, \( \psi_i \), that can contribute to an increase in the emissions intensity of the firm’s capacity portfolio. As the emissions intensity of the dirty type decreases, its production cost decreases. As the dirty type production cost decreases, its share of the firm’s optimal capacity portfolio increases. Given that the dirty type’s emissions intensity is greater than the clean types, an increase in the dirty types’ share of the firm’s optimal portfolio results in a greater emissions intensity of that portfolio. When the conditions in Corollary 4 hold, this second order effect through technology share dominates the direct benefit of improvement to the dirty type’s emissions intensity. As a result, the overall emissions intensity of the firm’s optimal capacity portfolio increases.

With density functions in the denominator of the RHS terms (i.e., the RHS of both conditions is likely to be large), the conditions in Corollary 4 would hold in most practical settings. This result, therefore, has important policy implications. Under the conditions noted in Corollary 4a and b, motivating firms to improve the emissions intensity of currently available dirty technologies could have the unintended consequence of inhibiting future investment in cleaner alternatives. This would lead to a relative increase in emissions intensity of a firm’s optimal portfolio. Improvements in the emissions intensity of the clean type never result in an increase in the emissions intensity of the portfolio. This is clear intuitively, and in Corollary 4 as the RHS of both conditions are negative if the change were made to the emissions intensity of the clean type.

In addition to the potential adverse effects of improvements in type 1 emissions intensity, market price can also impact the emissions intensity of the optimal portfolio.

**Corollary 5.** Under an emissions tax regime, the emissions intensity of the firm’s optimal portfolio increases monotonically in price if the clean technology is preferred in merit order, and decreases monotonically in price if the dirty technology is preferred.

It is evident from Proposition 1 that, given an interior solution, the preferred type is perfectly inelastic to changes in unit price, \( p \). The optimal type [2] capacity, however, increases monotonically in price. As a consequence, the technology share of the less preferred type, \( \psi_{[2]} \), increases. As \( \psi_{[2]} \) increases, the emissions intensity of capacity converges toward the emissions intensity of the less preferred type. Exogenous changes in price, therefore, also affect the emissions intensity of a firm’s optimal portfolio. This has significant policy implications.
In settings where carbon leakage is a threat, such as under EU-ETS regulation, policy-makers have debated implementing border adjustments (effectively, a carbon tariff on imports into the region). The market price in an emissions regulated region would be greater in a state of the world with border adjustments than a state without (a consequence of increased entry). Therefore, by Proposition 1, policy-maker’s decision regarding the implementation of border adjustments will affect the emissions intensity of capacity portfolios within the region, with the emissions intensity of those portfolios increasing as a result of the border adjustment if the dirty type is less preferred in merit order, and decreasing as a result of the border adjustment if the clean type is less preferred in merit order. In either case, by Proposition 1, the border adjustment decision would have unexpected consequences on emissions, unless policy-makers accounted for these portfolio affects.

4. Technology Choice under Cap-and-trade Regulation

The set-up for the cap-and-trade model resembles that of the emissions tax model in Section 3 except the constant tax rate, $\tau$, (known ex-ante) is replaced by an uncertain emissions price, $\tilde{e}$. Therefore, we treat $\tilde{e}$ as exogenous to the firm’s technology choice and production decisions as no firm possesses the market power to significantly influence the emissions allowance market price.

4.1. Cap-and-trade model

We denote decisions and objectives in the cap-and-trade setting with a $\hat{\cdot}$ – firms choose quantities, $\hat{q}_i$, in stage two to maximize operating margin, $\hat{\pi}$; and they choose capacities, $\hat{K}_i$, in the first stage to maximize expected profit, $\hat{\Pi}$. In addition to assuming that the firm is a price-taker, in the cap-and-trade setting we assume demand and emissions price distributions are independent, and we abstract from the option to financially hedge against emissions price uncertainty, focusing instead on the operational decisions of technology choice and capacity investment.

Second stage problem. Within the second stage of the cap-and-trade setting, the firm solves the following operating margin maximizing problem:

$$\max_{\hat{q}_1, \hat{q}_2} \hat{\pi} (\hat{K}_1, \hat{K}_2, d, e) = \max_{\hat{q}_1, \hat{q}_2} \left( \sum_{i=1}^{2} (p - b_i - \alpha_i e) \hat{q}_i \right) - r \left( d - \sum_{i=1}^{2} \hat{q}_i \right)$$

$$= \max_{\hat{q}_1, \hat{q}_2} \left( \sum_{i=1}^{2} \eta_i(e) \hat{q}_i \right) - rd$$

s.t. $0 \leq \hat{q}_i \leq \hat{K}_i, \forall i$

$$\sum_{i=1}^{2} \hat{q}_i \leq d.$$

Emissions costs are applied to all of the firm’s emissions – i.e., we assume full auctioning, which is the end-state of the EU-ETS (planned for January 2013) and other cap-and-trade legislation.
Define the set of feasible stage two technology types \( \tilde{N} \subseteq N \) as \( \{ i : p + r - b_i - \alpha_i e \geq 0 \} \), with this condition ensuring that only types profitable to utilize in the second stage belong to the set \( \tilde{N} \). With the uncertain demand and emissions price resolved, the second stage is deterministic and can be solved through greedy allocation. As in the emissions tax setting, \( \xi(e, i) \) orders the elements of the vector \( \hat{K}(e) \) from least to greatest operating cost, and we continue to use the notation \([i] \) for \( \xi(e, i) \). This yields operating margin maximizing quantities

\[
\hat{q}_{[i]} = \begin{cases} 
\min \left( \hat{K}_{[i]}, \left( d - \sum_{k=1}^{i-1} \hat{K}_{[k]} \right)^+ \right) & \forall i \in \tilde{N} \\
0 & \forall i \in N \setminus \tilde{N}.
\end{cases} \tag{5}
\]

**First stage problem.** The first stage cap-and-trade decision is made under uncertainty, both with respect to demand and production cost, with the firm choosing capacities to maximize profits:

\[
\max_{\hat{K}_1, \hat{K}_2} \hat{\Pi} \left( \hat{D}, \hat{e} \right) = \max_{\hat{K}_1, \hat{K}_2} \mathbb{E}_{\hat{D}, \hat{e}} \left[ \hat{\pi} \left( \hat{K}_1, \hat{K}_2, \hat{D}, \hat{e} \right) \right] - \sum_{i=1}^{2} \beta_i \hat{K}_i \tag{6}
\]

\[
\text{s.t.} \hat{K}_i \geq 0, \forall i.
\]

To avoid either type being trivially excluded from the firm’s optimal capacity portfolio due to being unprofitable in expectation, we assume \( \mathbb{E}_{\hat{e}} [\eta_i(\hat{e})] - \beta_i > 0 \), for both types.

Since production cost is uncertain given the stochastic emissions price, merit order is also uncertain in the first stage. Therefore, we partition the support of \( \hat{e} \) into merit order intervals \( \Omega_j \), \( j \in \Theta = \{1, \ldots, \theta \} \) with the boundaries of each interval determined in three ways: by the lower and upper limit of the support for \( \hat{e} \) (defining the lower bound of interval \( j = 1 \) and the upper bound of \( j = \theta \)), by changes in merit order (i.e., the production cost crossing point for two types), and by changes in the membership of \( \tilde{N} \) (i.e., where a type’s production cost crosses the threshold \( p + r \)). Let \( \Omega_j \) represent the interval over \( \hat{e} \) where the \( j \)th merit ordering holds. In general, the number of merit order permutations, \( \theta \), can be at most \( 1 + \binom{n+1}{2} \). Therefore, \( \theta \leq 4 \) in our 2-type setting.

Define \( \Omega_1 \) as the merit order interval where type 1 capacity is preferred and define \( \Omega_2 \) as the interval where type 2 capacity is preferred. Further, define \( \Omega_3 \) as the merit order interval where one type is unprofitable to operate, with \( \Omega_{3,i} \) noting type \( i \) as the profitable type (i.e., \( \tilde{N} = \{ i \} \) for all \( e \in \Omega_{3,i} \)). Finally, define \( \Omega_4 \) as the interval where neither type is profitable to operate. If type \( i \) is dominated in merit order over the support of \( \hat{e} \), then \( \Omega_i \) and \( \Omega_{3,i} \) are empty. Note that any or all of the merit order intervals appearing in these FOC (i.e., \( \Omega_1 \), \( \Omega_2 \), and \( \Omega_3 \)) can be empty.

Concavity can be proven directly through the Hessian as illustrated within Appendix 1. The cross-partial is non-positive. As a consequence, the upper bound of \( \hat{K}_i \) in the cap-and-trade setting, defined here as \( \hat{K}_i^C \), is determined when \( \hat{K}_{-i} = 0 \). Therefore, \( \hat{K}_i^C = F_D^{-1} \left( 1 - \frac{\beta_i}{\eta_i(\Omega_i) + \eta_i(\Omega_{-i}) + \eta_i(\Omega_{3,i})} \right) \).
4.2. Cap-and-trade with a dominant second stage technology

Imagine a cement manufacturer planning their long-term capacity portfolio. In developing their capacity, they decide between standard kilns, kilns fitted with CCS$^3$, or a combination of the two. CCS kilns increase operating costs (i.e., $b_2 > b_1$), but vastly reduce emissions costs (i.e, $\alpha_2 e < \alpha_1 e$). Under such a setting, standard kilns would dominate merit order unless it was possible for the emissions allowance price to be sufficiently high that the firm would prefer to dispatch the CCS technology. However, even if CCS were dominated in merit order, the firm may still opt to include it within their capacity portfolio if its per unit investment cost were lower than that of traditional kilns ($\beta_2 < \beta_1$), and sufficiently low to offset its operating margin disadvantage. We address that setting here, where one technology dominates merit ordering over the support of $\hat{e} - i.e., there exists an $i$ where $\Pr(b_i + \alpha_i \hat{e} \leq b_{-i} + \alpha_{-i} \hat{e}) = 1$.

Solution. We summarize the solution to the cap-and-trade setting when a type $i$ dominates the merit order with the following proposition:

**Proposition 2.** Assume cap-and-trade regulation, and assume type $i$ dominates merit order, i.e., $\Pr(b_i + \alpha_i \hat{e} \leq b_{-i} + \alpha_{-i} \hat{e}) = 1$, then the vector of optimal capacities, $\hat{K}^*$ is characterized by

$$
\hat{K}_i^* = \begin{cases} 
K_i^C & \text{if } \beta_{-i} \geq \beta_i \frac{\eta_{-i}(\Omega_i)}{\eta_{i}(\Omega_i) + \eta_{i}(\Omega_{-i}) - \eta_{-i}(\Omega_i)} \\
0 & \text{if } \eta_{-i}(\Omega_i) - \beta_{-i} \geq \eta_{i}(\Omega_i) + \eta_{i}(\Omega_{-i}) - \beta_i \\
F_{D}^{-1}\left(1 - \frac{\beta_{-i} - \beta_{-i}}{\eta_{i}(\Omega_i) + \eta_{i}(\Omega_{-i}) - \eta_{-i}(\Omega_i)}\right) & \text{otherwise}, 
\end{cases}
$$

and

$$
\hat{K}_{-i}^* = \begin{cases} 
0 & \text{if } \beta_{-i} \geq \beta_i \frac{\eta_{-i}(\Omega_i)}{\eta_{i}(\Omega_i) + \eta_{i}(\Omega_{-i}) - \eta_{-i}(\Omega_i)} \\
K_{-i}^C & \text{if } \eta_{-i}(\Omega_i) - \beta_{-i} \geq \eta_{i}(\Omega_i) + \eta_{i}(\Omega_{-i}) - \beta_i \\
F_{D}^{-1}\left(1 - \frac{\beta_{-i}}{\eta_{i}(\Omega_i) + \eta_{i}(\Omega_{-i}) - \eta_{-i}(\Omega_i)}\right) - \hat{K}_i^* & \text{otherwise}. 
\end{cases}
$$

As under emissions tax regulation described in Section 3, solution fractiles in Proposition 2 consist of newsvendor differences. Note that $\hat{K}_{[1]}^*$ resembles the solution for the merit order dominant type under an emissions tax setting, except that operating margins in the cap-and-trade setting are expectations based on the uncertainty of $\hat{e}$. Similarly, the solution for $\hat{K}_{[2]}^*$ mirrors that of the merit order dominated type in Section 3. Despite dominating in the second stage, type $[1]$ technology would not be included within the optimal capacity portfolio if expected per unit profit margin increases in merit order. In such a case, the operating margin advantage of the merit order dominant technology would be more than offset by its investment cost disadvantage – i.e., expected profit would be less and investment cost would be greater than the merit order dominated technology – making it an unattractive option. As $\tilde{\eta}_{[2]}(\Omega_{[1]}) \leq \tilde{\eta}_{[1]}(\Omega_{[1]})$ and $\tilde{\eta}_{[1]}(\Omega_{[3]}) > 0$, the RHS of the boundary condition for type $[2]$ technology is less than $\beta_{[1]}$. Therefore, this boundary condition refines the intuition as to when the dominated type will be included within the firm’s optimal capacity portfolio (recall the necessary condition $\beta_{[1]} > \beta_{[2]}$).

---

$^3$ CCS is being piloted within the cement industry today, and is expected to be commercially available by 2020.
4.3. Cap-and-trade with no dominant second stage technology

Returning to the cement firm developing its capacity with standard and CCS kilns, if emissions price realizations could be sufficiently high for the firm to prefer to dispatch CCS technology (i.e., \( \Pr (e > \frac{b_2 - b_1}{a_2 - a_1}) > 0 \)), then merit order would change over the support of \( \tilde{e} \). In such a case, there would be an interval over \( \tilde{e} \) where the firm would prefer to dispatch standard technology, and an interval where it would prefer to dispatch CCS technology.

Solution. Capacity solutions in the cap-and-trade setting where no type dominates are symmetric for \( \hat{K}_1 \) and \( \hat{K}_2 \) and are characterized by the following proposition,

**Proposition 3.** Under cap-and-trade regulation where neither type dominates merit order, the optimal vector of capacities is characterized by

\[
\hat{K}^*_i(\hat{K}^*_1) = \begin{cases} 
0 & \text{if } \mathbb{E}_e [\eta_i(\hat{e})] - \beta_i \leq F_D(\hat{K}^*_1) \left( \eta_i(\Omega_{-i}) - \tilde{\eta}_{-i}(\Omega_i) \right) \\
\hat{K}^C_i & \text{if } \mathbb{E}_e [\eta_{-i}(\hat{e})] - \beta_{-i} \leq F_D(\hat{K}^*_1) \left( \tilde{\eta}_{-i}(\Omega_i) - \eta_i(\Omega_{-i}) \right) \\
F_D^{-1} \left[ (1 - \left( \frac{\beta_i - \beta_{-i}}{\eta_i(\Omega_{-i}) + \eta_{-i}(\Omega_i)} \right) \tilde{\eta}_{-i}(\Omega_i) + \eta_i(\Omega_{-i}) - \eta_{-i}(\Omega_i)) \right] & \text{otherwise.}
\end{cases}
\]

The terms within the left hand parentheses of the fractile in the interior solution are identical to the interior solution for type 1 capacity when it dominates merit order as characterized in Proposition 2. The second term of the interior solution is always positive. Therefore, the dominant interior solution given in Proposition 2 provides the upper bound for interior solutions in the mixed merit order setting. The ratio of the last term within the fractile of the interior solution is the expected difference in operating margin for each type when type 2 is favored (the numerator) and when type 1 is favored (the denominator). As such, this ratio can be thought of as a substitution effect; a relatively greater expected margin for type \( i \) capacity leads to a relatively smaller investment in type \( -i \) capacity. The RHS of the boundary condition equates to the \( \hat{K}^C_i \) fractile multiplied by the difference in expected operating margin for each type when the opposite type is preferred in merit order. Note that the difference in expected operating margin in this condition will be non-positive for either the type \( i \) or type \( -i \) boundary condition. Therefore, since \( \mathbb{E}_e [\eta_i(\hat{e})] - \beta_i > 0 \) by assumption, this condition implies that at least one type will be included within the optimal capacity portfolio.

Using the symmetric reaction curves \( \hat{K}^*_i(\hat{K}^*_1) \) defined in Proposition 3, numerical solutions can be obtained through an iterative procedure that converges globally to the optimal solution under the following strict concavity (SC) condition: \( f_D(x) > 0, \forall x \in [0, \max\{\hat{K}^C_1, \hat{K}^C_2\}] \).

**Proposition 4.** Assume SC\(^4\). Given any feasible solution \( \hat{K}^0_i \) and \( \hat{K}^0_{-i} \), the sequence \( \hat{K}^t \) defined by \( \hat{K}^{t+1}_i = \hat{K}^*_i(\hat{K}^t_{-i}) \) and \( \hat{K}^{t+1}_{-i} = \hat{K}^*_i(\hat{K}^t_i) \), \( t = \{0, 1, 2, \ldots\} \) converges to \( \hat{K}^*_i \) and \( \hat{K}^*_i \).

\(^4\) In settings where SC does not hold, Proposition 4 can still be used to solve the original problem by subtracting \( \epsilon(\hat{K}^2_i + \hat{K}^2_{-i}) \), \( \epsilon > 0 \) from the objective function given by (6). As explained in Appendix 1, this gives rise to a strictly concave objective function whose solution converges to the solution of (6) as \( \epsilon \to 0 \).
4.4. Comparing expected profit across emissions regulation regimes

Given that proponents of a carbon tax argue that the uncertainty of emissions price under cap-and-trade would erode firm profits, it is interesting to compare the profitability of a firm under cap-and-trade to a firm under an emissions tax. We do so here and find the opposite to be true.

**Proposition 5.** Assume $\mu_\varepsilon \leq \tau$. A firm’s expected profit is greater under a cap-and-trade regime than it is under an emissions tax regime, $\hat{\Pi}(\tilde{D},\tilde{e}) \geq \Pi(\tilde{D},\tau)$.

Thus, assuming that the expected cap-and-trade price, $\mu_\varepsilon$, is no greater than the emissions tax, $\tau$, the added uncertainty that the firm is exposed to under a cap-and-trade setting does not erode value as proponents of a carbon tax argue. Rather, this added uncertainty creates value. Firm’s under a cap-and-trade regime are equipped with the real option to leave demand unserved, which they would exercise if emissions costs were sufficiently great that it becomes unprofitable to satisfy demand. A firm under cap-and-trade that ignores this option and always meets demand or utilizes all of its capacity would earn the same expected profit as a firm under an emissions tax since $\mu_\varepsilon = \tau$. By choosing to exercise its production option, leaving demand unserved when emissions price is sufficiently high to make it unprofitable to produce, a firm avoids worst-case emissions price outcomes and increases its expected profit.

The firm’s ability to effectively right censor the emissions prices it is exposed to through its production option has two opposing effects. First, avoiding the least favorable emissions price outcomes in this manner means that the expected profit margin per unit of demand that the firm does serve is greater under cap-and-trade than it is under an emissions tax. Second, and partially off-setting the unit margin affect, the firm’s choice not to serve demand when the emissions price realization makes it unprofitable to do so implies that the total demand served will be less under cap-and-trade than it is under an emissions tax. These effects act in opposite directions on a firm’s total profits, but the margin effect dominates. Therefore, the option to leave demand unserved creates value for the firm whenever the emissions price distribution has support over the interval where it would be unprofitable to operate capacity within the firm’s portfolio, i.e., when $\Pr(\tilde{e} \geq \frac{p^* + r - b[2]}{\sigma[2]}) > 0$ for interior solutions, and when $\Pr(\tilde{e} \geq \frac{p^* + r - b[1]}{\sigma[1]}) > 0$ for boundary solutions. This result runs counter to arguments put forward by carbon tax proponents who claim that greater emissions price uncertainty under cap-and-trade will erode firm profitability. As a consequence, this result has policy implications in the ongoing cap-and-trade versus carbon tax debate.

5. Numerical Analysis

We numerically explore the impact of the capacity decisions in Propositions 3 and 1 on key metrics. In addition to the capacity solutions, we consider clean technology share, $\psi_2$, as well as the expected
emissions intensity of production, $\nu_q$, as previously defined. We use boldface notation (such as $K_i^*$ and $q_i^*$) to indicate general decisions and metrics, rather than regime-specific solutions.

In addition to these metrics, we also explore impacts on expected profit, which is defined by (6) for the cap-and-trade setting and by (3) for an emissions tax regime. We consider total expected emissions, $\varepsilon = \mathbb{E}_{D,\tilde{e}}[\alpha_1 q_1^* + \alpha_2 q_2^*]$, and total expected production, $\varphi = \mathbb{E}_{D,\tilde{e}}[q_1^* + q_2^*]$, to compare environmental impact and output across regime type and specific parameter values. We also refer in our discussion to service level, $\mathbb{E}_{D,\tilde{e}}[\alpha_1 q_1^* + \alpha_2 q_2^*]$.

We consider a cement manufacturer developing a portfolio from two technologies: standard kilns (type 1) and CCS kilns (type 2). All parameters used within these numerics are derived from our field research\(^5\), and publicly available sources. The motivation behind parameter values is described within online Appendix O.1, with values summarized here. Type 1 parameters are $\beta_1 = 10.1$, $b_1 = 43.6$, and $\alpha_1 = 0.700$, while type 2 parameters are $\beta_2 = 14.6$, $b_2 = 55.0$, and $\alpha_2 = 0.075$. The distribution of emissions allowance price in the cap-and-trade setting is assumed to be lognormal with an average emissions allowance price of $\mu_\tilde{e} = 25$ and a standard deviation of emissions allowance prices of $\sigma_\tilde{e} = 15$. Within the tax setting, we assume the tax rate is equivalent to mean emissions allowance price under cap-and-trade. Therefore, $\tau = 25$. The distribution of demand is also assumed to be lognormal, but we normalize average demand to $\mu_\tilde{D} = 1000$ while preserving our focal firm’s estimated coefficient of variation (CoV = 0.175) with $\sigma_\tilde{D} = 175$. Lastly, $p = 90$ and $r = 0$.

5.1. Baseline levels

Table 1 provides the results for each of the capacity and performance metrics described above.

<table>
<thead>
<tr>
<th>Regime</th>
<th>Capacity Metrics</th>
<th>Expected Performance Metrics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$K_1^<em>$ $K_2^</em>$</td>
<td>$\psi_2$</td>
</tr>
<tr>
<td>Cap-and-trade</td>
<td>942 119 1,061</td>
<td>11.2%</td>
</tr>
<tr>
<td>Emissions tax</td>
<td>1,053 0 1,053</td>
<td>0.0%</td>
</tr>
<tr>
<td>No Regulation</td>
<td>1,128 0 1,128</td>
<td>0.0%</td>
</tr>
</tbody>
</table>

Table 1 Profit-maximizing capacity and performance metrics for a traditional versus CCS kiln decision.

Comparing both regimes to the unregulated case, it is clear that emissions regulation is likely to be extremely costly to the cement sector, eroding nearly 50% of profits. The magnitude of this effect aligns with with cement executive expectations that production costs will increase by 40% under the full auctioning of emissions allowances planned to begin with Phase III of the EU-ETS.

Looking specifically at the tax regime, type 2 technology is preferred in merit order (i.e., $\eta_1(\tau) < \eta_2(\tau)$). However, profit margin increases in merit order, $\eta_1(\tau) - \beta_1 > \eta_2(\tau) - \beta_2$, so the Proposition 1 boundary condition holds. As a consequence, $K_2^* = 0$ and $K_1^* = \bar{K}_1^T = 1053$. In the cap-and-trade

\(^5\) Citation for field research suppressed in the interest of blind review.
setting, conditions for an interior solution hold. Therefore, CCS is included in the the cap-and-trade portfolio with a share of 11.2%. This result runs counter to the popular belief that the cost certainty of a carbon tax regime will incentivize greater investment in clean technology (e.g., Shapiro 2010).

Although the firm would optimally invest in less total capacity under a tax regime, expected production is greater. With the distribution of demand identical between regime-type, this implies that service level is greater under the emissions tax regime than under cap-and-trade in this context (with unit service levels of 95.2% and 93.7%, respectively). This results from the possibility under cap-and-trade that emissions price could be great enough that the firm would opt not to operate standard kilns – i.e., \( \Pr(e \in \Omega_{3,2}) = 0.021 \) – thus lowering its overall expected service level.

Emissions in this setting are 11.7% more under an emissions tax than under a cap-and-trade regime. This results, in part, from the lower production under the cap-and-trade regime. It also results in part from the decreased emissions intensity of production. Separating these effects, we note a volume effect of 10.48 emission tons and a capacity mix effect of 59.08 emission tons per 1000 tons of expected cement demand. Roughly 85% of the difference in emissions resulted from the firm’s technology choice.

The firm earns greater expected profit under the cap-and-trade than the tax regime. The magnitude of the difference is minimal because the probability that the firm would opt not to operate is small. But what is interesting here is that the cap-and-trade regime generated greater total profit (per Proposition 5). This runs counter to the arguments put forward by proponents of a carbon tax – that the added uncertainty of emissions prices under cap-and-trade will destroy value.

Finally, the policy-maker in this context would face competing social welfare effects: lower emissions and greater expected firm profits under cap-and-trade regulation relative to emissions tax regulation, but also lower expected production (implying lower consumer surplus in this context).

5.2. Sensitivity to investment cost and emissions intensity

We explore the sensitivity of decisions and metrics to changes in parameter values through comparative statics. We evaluated the elasticity of key metrics in the cap-and-trade setting to each parameter and report these in Table A3.1 in online Appendix 3. Of particular interest, in this context, a 1% increase in the production cost of the dirty type led to nearly a 19% decrease in emissions and to a slight increase in production, \( \varphi \), of 0.5%. This counter-intuitive increase in production resulted from second order effects. Increased type 1 costs led to an increase in clean technology share, \( \psi_2 \). This implied that a greater percentage of the firm’s capacity would operate when emissions allowance price fell within \( \Omega_{3,2} \), the region in which the firm would opt to leave dirty technology idle. With \( \Pr(\Omega_{3,2}) \) sufficiently positive in this context, expected production increased.

\[ \varepsilon_c - \varepsilon_t = \left( \frac{\varphi_1 - \varphi_2}{\psi_1} \right) \varepsilon_t + \left( \frac{\varphi_1 - \varphi_2}{\psi_2} \right) \varepsilon_c. \]

Focusing on the LHS, the left term represents the volume effect and the right term represents the mix effect.

\footnote{Let \( c \) and \( t \) denote cap-and-trade and tax regimes, respectively. It can be shown that \( \left( \frac{\varphi_1 - \varphi_2}{\psi_1} \right) \varepsilon_t + \left( \frac{\varphi_1 - \varphi_2}{\psi_2} \right) \varepsilon_c = \varepsilon_c - \varepsilon_t. \) Focusing on the LHS, the left term represents the volume effect and the right term represents the mix effect.}
Figures 1a and 1b explore the effect of a change in the investment cost of clean technology, $\beta_2$, on clean technology share.

The effect in the tax setting was dramatic in the neighborhood where the boundary condition determining the dominance of type 1 over type 2 was just binding. At this point, a very slight further reduction of $\beta_2$ led to an increase in clean technology share of just over 60%. Thus, in the neighborhood of this boundary, a relatively small subsidy or discount would have a profound effect. Figure 1a shows that this change in share is more tapered under cap-and-trade. Figure 1a and 1b levels are reported in Tables A3.2a and b in Appendix O.2.

Elasticities also indicated that Corollary 4 held in the cap-and-trade setting in this context: a 1% increase in the emissions intensity of type 1 technology in the cap-and-trade setting led to nearly a 4% decrease in the emissions intensity of production, $\nu_q$. This resulted from the increase in type 1 emissions intensity leading to a 45% increase in clean technology share, $\psi_2$. This result has implications when considering the potential impact of improvements in emissions intensity. Should a firm make improvements by reducing the emissions intensity of their installed capacity (standard kilns in this case), it may have the unintended effect of reducing the adoption of clean technology in the future (such as CCS kilns). This represents a possible temporal trade-off in emissions abatement; incremental improvements to emissions intensive technologies can, in some cases, inhibit technology adoption that would have more dramatic emissions reduction effects. Figures 2a and 2b provide further insight into this effect.

Improvements to $\alpha_1$ (decreases) led to a degradation of clean technology share within the optimal capacity portfolio due to the lower emissions cost of the dirty technology. This resulted in an increase in emissions intensity of production, $\nu_q$ until the optimal portfolio consisted only of dirty technology. When clean technology share was zero, $\psi_2 = 0$, the emissions intensity of production becomes equivalent to the emissions intensity of the dirty type, $\alpha_1$. Only from that point did further
improvements in $\alpha_1$ lead to a reduction in the emissions intensity of production. This holds at a variety of levels of clean technology investment cost, $\beta_2$, in this context. Levels of the baseline (bold) curves are provided in Tables A3.3a and b within online Appendix O.2.

6. Implications and Conclusions

In exploring a firm’s discrete technology choice and capacity portfolio under emissions regulation, we have identified a number of potential unintended consequences that can arise as a consequence of emissions legislation. Results from this research have implications for several stakeholders, with insights for capacity owners, technology producers and policy-makers.

**Capacity owners.** The problem we study focuses on the capacity owner’s challenge of maximizing profits by trading off investment cost, operating cost, and emissions efficiency – a challenge faced by firms under emissions regulation such as in the power generation, cement, steel, and pulp-and-paper sectors, among others. We study the problem in both a cap-and-trade and an emissions tax setting, as cap-and-trade regulation already exists and the political feasibility of a tax regime seems to be on the rise. Propositions 1, 2 and 3 characterize conditions where the optimal portfolio should consist of a single technology type, and where it should consist of a mix of technologies. Further, assuming that the emissions tax is no less than the expected price under cap-and-trade, Proposition 5 indicates that expected profits will be greater under cap-and-trade than under an emissions tax. To the extent that firms can have a voice in policy selection, this result suggests that they should recognize the real option embedded in their production decision and favor a cap-and-trade regime.

**Technology producers.** While not the focus of our work, there are insights to be gleaned by technology producers within our results. The boundary conditions defined in Propositions 1, 2 and 3 can provide guidance regarding the feasibility of a new technology in the marketplace. The interior solutions presented within these propositions could be helpful in developing projections for
long-term market share for technology segments (i.e., sets of competing, like-type technologies). Further, the optimal portfolio’s high level of sensitivity to parameter values, and in particular, investment cost, points to the technology producer’s ability to potentially impact adoption with relatively minor price discounts. Technology share curves in Section 5 were substantially steeper under the carbon tax regime than under cap-and-trade, which suggest that price competition between producers of clean and dirty technologies could be stiffer under the former regime.

Policy-makers. In many cases, such as with the EU-ETS, policy-makers must design emissions legislation for a set of heterogeneous industries. The sensitivity of the optimal portfolio suggests that the policy-maker could use subsidies to fine-tune the regulation within a particular industry where the general legislation does not have the desired effect. Corollary 1 indicates that investment subsidies would provide a more sensitive lever in such cases than production subsidies. However, as Corollary 2 indicates, such subsidies can conditionally increase the emissions potential of the optimal capacity portfolio, and therefore be counterproductive with respect to the legislation’s objective of emissions abatement. Our research also points to a number of nuances or unintended consequences that can result from emissions regulation. These contributions include: conditions when an increase in tax rate will decrease investment in clean technology (Corollary 3); conditions when an improvement in a dirty technology’s emissions intensity can lead to a negative long-term effect on the optimal portfolio’s emissions intensity (Corollary 4); and conditions when the optimal portfolio’s clean technology share would increase in price (Corollary 5). The latter is important where the threat of carbon leakage keeps price suppressed in an emissions-regulated industry, with the policy-maker weighing carbon tariff options. Finally, Proposition 5 counters prevailing arguments that an emissions tax would be more profitable for firms than a cap-and-trade regime due to the uncertainty in emissions price under cap-and-trade. The opposite is shown to be true and should be taken into account as the policy debate continues to unfold: the uncertainty in emissions price under cap-and-trade equips firms with a real option that increases expected profits.

References


**Appendix 1: Proofs**

Proof of concavity in emissions tax setting. Concavity can be proven directly through the Hessian, with the Hessian in the emissions tax setting, $H(P_i)$, defined by

$$H(\Pi) = \begin{bmatrix}
\frac{\partial^2 \Pi}{\partial K_{[1]}^2} & \frac{\partial^2 \Pi}{\partial K_{[1]} \partial K_{[2]}} \\
\frac{\partial^2 \Pi}{\partial K_{[1]} \partial K_{[2]}} & \frac{\partial^2 \Pi}{\partial K_{[2]}^2}
\end{bmatrix},$$

where

$$\frac{\partial^2 \Pi}{\partial K_{[1]}^2} = -f(K_{[1]})(\eta_{[1]} - \eta_{[2]}) - f(K_{[1]} + K_{[2]})\eta_{[2]},$$

(7)

$$\frac{\partial^2 \Pi}{\partial K_{[2]}^2} = -f(K_{[1]} + K_{[2]})\eta_{[2]},$$

and

$$\frac{\partial^2 \Pi}{\partial K_{[1]} \partial K_{[2]}} = -f(K_{[1]} + K_{[2]})\eta_{[2]}.$$
The first order leading principle minor is defined by (7) and is non-positive given that \( \eta[i] > \eta[2] > 0 \) by merit ordering and the assumption that \( \eta[i] - \beta[i] > 0 \). This principle minor is strictly negative when \( f(K[i]) > 0 \) and/or \( f(K[i] + K[2]) > 0 \).

The second order leading principle minor is defined by the determinant of the Hessian, \( |H(\Pi)| \), where

\[
|H(\Pi)| = \left(f(K[i] + K[2])\eta[2]\right) \left|f(K[i])(\eta[i] - \eta[2])\right|.
\]

(8)

The second leading principle minor is clearly non-negative, and is strictly positive given \( f(K[i]) > 0 \) and \( f(K[i] + K[2]) > 0 \). Thus, the matrix \( H(\Pi) \) is semi-negative definite. It is negative definite whenever \( f(K[i]) > 0 \) and \( f(K[i] + K[2]) > 0 \). Therefore, \( \Pi \) is concave in capacities \( K[i] \) and \( K[2] \), and strictly concave in \( K[i] \) and \( K[2] \) when \( f(K[i]) > 0 \) and \( f(K[i] + K[2]) > 0 \). □

**Proof of Proposition 1.** Substituting (2) into the firm’s objective in the emissions tax setting given by (3), then transforming the min arguments with positive parts yields

\[
\max_{K_1, K_2} \Pi(\bar{D}, \tau) = \sum_{i=1}^{2} E \left[ \eta[i](\tau) \left( \bar{D} - \sum_{k=1}^{i} K[k] \right)^+ \left( \bar{D} - \sum_{k=1}^{i-1} K[k] \right)^+ \right] - E[r\bar{D}] - 2 \sum_{i=1}^{2} \beta[2]K[i]
\]

s.t. \( K[i] \geq 0, \forall i \).

Solving (9) for FOCs yields

\[
\frac{\partial \Pi}{\partial K[i]} = \left(1 - F_D^{-1}\left(\sum_{k=1}^{i} K[k]\right)\right) \eta[i] - \sum_{i=1}^{2} \left[ F_D\left(\sum_{k=1}^{i} K[k]\right) - F_D\left(\sum_{k=1}^{i-1} K[k]\right) \right] \eta[i] - \beta[i] = 0.
\]

(10)

Solving (10) for \( i = 2 \) yields total installed capacity,

\[
\sum_{k=1}^{2} K^*[k] = F_D^{-1}\left(1 - \frac{\beta[2]}{\eta[2](\tau)} \right).
\]

(11)

Solving (10) for \( i = 1 \) yields the solution for the preferred type,

\[
K^*[1] = F_D^{-1}\left(1 - \frac{\beta[1] - \beta[2]}{\eta[1](\tau) - \eta[2](\tau)} \right).
\]

(12)

Type [2] capacity is the difference between (11) and (12). Boundary conditions follow from the requirement that the argument in (12) be non-negative and the requirement that the difference between (11) and (12) be non-negative. □

**Proof of Corollary 1.** To simplify analysis, we differentiate with respect to \( \beta_2 \) and \( b_2 \), understanding that a per unit subsidy would decrease either parameter (depending on whether it was an investment or production subsidy, respectively). The Corollary 1a and b result follows directly from the solutions characterized within Proposition 1.

With respect to changes in the unsubsidized capacity type (Corollary 1a),

\[
\frac{\partial K[1]}{\partial \beta[2]} = \frac{\partial K[1]}{\partial b[2]} = f_D\left(1 - \frac{\beta[1] - \beta[2]}{\eta[2] + \alpha[2] - \alpha[1]} \right) \left(1 - \frac{\beta[1] - \beta[2]}{b[2] + \alpha[2] - \alpha[1]} \right) \geq 0.
\]
which holds only under the condition given in Corollary 2b.

Capacity \( K_i, i \in \{1, 2\} \), increases in \( \beta_{-i} \) at a greater rate than it increases in \( b_{-i} \), regardless of whether type \( i \) is preferred in merit order. Therefore, capacity of the unsubsidized type, \( K^*_i \), decreases at a greater rate in a per unit subsidy, \( \delta \) applied to \( \beta_{-i} \), than a subsidy \( \delta \) applied to \( b_{-i} \).

\[
\frac{\partial K_{[1]} - \partial K_{[2]}}{\partial \beta_{[1]}} = \frac{1}{f_D \left( 1 - \frac{\eta_{[1]} - \eta_{[2]}}{\eta_{[1]}(\tau) - \eta_{[2]}(\tau)} \right)} \left( 1 - \frac{\beta_{[1]} - \beta_{[2]}}{\beta_{[1]} - \beta_{[2]}} \right) \left( \eta_{[1]}(\tau) - \eta_{[2]}(\tau) \right) \leq 0,
\]

and

\[
\frac{\partial K_{[1]} - \partial K_{[2]}}{\partial b_{[1]}} = \frac{1}{f_D \left( 1 - \frac{\eta_{[1]} - \eta_{[2]}}{\eta_{[1]}(\tau) - \eta_{[2]}(\tau)} \right)} \left( 1 - \frac{\beta_{[1]} - \beta_{[2]}}{\beta_{[1]} - \beta_{[2]}} \right) \left( \eta_{[1]}(\tau) - \eta_{[2]}(\tau) \right) \leq 0,
\]

Like-type capacity, \( K_i, i \in \{1, 2\} \), decreases in \( \beta_i \) at a greater rate than it decreases in \( b_i \), regardless of whether it is preferred in merit order (i.e., type [1]) or not (i.e., type [2]).

Total capacity when type 1 is preferred in merit order is 

\[
\frac{\partial K^*_{1} + K^*_2}{\partial \beta_{[2]}} < \frac{\partial K^*_{1} + K^*_2}{\partial b_{[2]}} = -1 < -\frac{\beta_{2}}{\eta_{2}(\tau)},
\]

which are both negative. Therefore, type 2 capacity increases at a greater rate in a per unit investment subsidy, \( \delta \), than a per unit production subsidy, \( \delta \). □

**Proof of Corollary 2.** When type 1 is preferred in merit order, increases in an investment subsidy \( \delta \) are equivalent to decreases of \( \delta \) in \( \beta_{[2]} \). Therefore, the total emissions potential of capacity increases in investment subsidy when

\[
\frac{\partial (\alpha_{[1]} K^*_{[1]} + \alpha_{[2]} K^*_{[2]})}{\partial \beta_{[2]}} = \alpha_{[1]} \left( -\frac{\beta_{[1]} - \beta_{[2]}}{\eta_{[1]}(\tau) - \eta_{[2]}(\tau)} \right) \left( \eta_{[1]}(\tau) - \eta_{[2]}(\tau) \right) \leq 0,
\]

which holds only under the condition given in Corollary 2a.

The total emissions potential of capacity increases in production subsidy when

\[
\frac{\partial (\alpha_{[1]} K^*_{[1]} + \alpha_{[2]} K^*_{[2]})}{\partial \beta_{[2]}} = \alpha_{[1]} \left( -\frac{\beta_{[1]} - \beta_{[2]}}{\eta_{[1]}(\tau) - \eta_{[2]}(\tau)} \right) \left( \eta_{[1]}(\tau) - \eta_{[2]}(\tau) \right) \leq 0,
\]

which holds only under the condition given in Corollary 2b. □
Proof of Corollary 3. The result follows directly from the derivatives,
\[
\frac{\partial K_1}{\partial \tau} = \frac{\beta_1 - \beta_2}{f_D} \left( 1 - \frac{\beta_1 - \beta_2}{\eta_1 - \eta_2} \right)^2, \tag{15}
\]
and
\[
\frac{\partial K_2}{\partial \tau} = -\frac{\beta_2 - \beta_1}{f_D} \left( 1 - \frac{\beta_2 - \beta_1}{\eta_1 - \eta_2} \right)^2. \tag{16}
\]
As a necessary condition for an interior solution \( \beta_1 > \beta_2 \). By the definition of merit ordering, \( \eta_1(\tau) > \eta_2(\tau) \).

When type 1 is preferred in merit order (i.e., it is type [1]), \( \alpha_1 > \alpha_2 \) and it follows that (15) is non-positive. When type 1 is not preferred in merit order (i.e., it is type [2]), \( \alpha_2 > \alpha_1 \) and it follows that (16) is non-positive. As a consequence the result of type 1 monotonically decreasing in \( \tau \) holds.

When type 2 is preferred in merit order, (i.e., it is type [1]), \( \alpha_1 < \alpha_2 \) and it follows that (15) is non-negative. When type 2 is not preferred in merit order (i.e., it is type [2]), \( \alpha_2 < \alpha_1 \) and the RHS of (16) is non-positive only when
\[
\frac{\alpha_1 - \alpha_2}{\alpha_2} \leq \frac{f_D \left( 1 - \frac{\beta_2 - \beta_1}{\eta_1 - \eta_2} \right)}{f_D \left( 1 - \frac{\beta_1 - \beta_2}{\eta_1 - \eta_2} \right)} \beta_2 (\eta_1 - \eta_2)^2,
\]
and the result follows. \( \square \)

Proof of Corollary 4. The proof of 5a, when the dirty technology is preferred, follows directly from the total derivative,
\[
\frac{d \nu_K}{d \alpha_1} = \frac{\partial \nu_K}{\partial K_1} \frac{\partial K_1}{\partial \alpha_1} + \frac{\partial \nu_K}{\partial K_2} \frac{\partial K_2}{\partial \alpha_1} < 0.
\]
Likewise, the proof of 5b, when the dirty technology is not preferred, follows from the total derivative,
\[
\frac{d \nu_K}{d \alpha_2} = \frac{\partial \nu_K}{\partial K_1} \frac{\partial K_1}{\partial \alpha_2} + \frac{\partial \nu_K}{\partial K_2} \frac{\partial K_2}{\partial \alpha_2} < 0. \quad \square
\]

Proof of Corollary 5. The result where price increases imply a decrease in the optimal portfolio’s emissions intensity follows from the total derivative
\[
\frac{d \nu_K}{d p} = \frac{\partial \nu_K}{\partial K_1} \frac{\partial K_1}{\partial p} + \frac{\partial \nu_K}{\partial K_2} \frac{\partial K_2}{\partial p} < 0. \tag{17}
\]
Solving (17) yields the condition \( \alpha_2 < \alpha_1 \), which holds if the dirty technology, type 1, is preferred (i.e., it is type [1]).

The result where price increases imply an increase in the emissions intensity of the optimal portfolio follows from the total derivative
\[
\frac{d \nu_K}{d p} = \frac{\partial \nu_K}{\partial K_1} \frac{\partial K_1}{\partial p} + \frac{\partial \nu_K}{\partial K_2} \frac{\partial K_2}{\partial p} > 0,
\]
which clearly yields a symmetric result. \( \square \)
Proof of concavity in emissions cap-and-trade setting. As with concavity in the emissions tax setting, concavity in the cap-and-trade setting can be proven directly through the Hessian,

\[ H(\hat{\Pi}) = \begin{bmatrix} \frac{\partial^2 \hat{\Pi}}{\partial K_1^2} & \frac{\partial^2 \hat{\Pi}}{\partial K_1 \partial K_2} \\ \frac{\partial^2 \hat{\Pi}}{\partial K_2 \partial K_1} & \frac{\partial^2 \hat{\Pi}}{\partial K_2^2} \end{bmatrix}. \] (18)

where

\[ \frac{\partial^2 \hat{\Pi}}{\partial K_i^2} = -f_D(\hat{K}_1 + \hat{K}_2)\eta_2(\Omega_i) - f_D(\hat{K}_1)(\eta_1(\Omega_i) - \eta_2(\Omega_i)) - f_D(\hat{K}_1 + \hat{K}_2)\eta_1(\Omega_2) - f_D(\hat{K}_1)\eta_1(\Omega_{3,1}), \]

\[ \frac{\partial^2 \hat{\Pi}}{\partial K_2^2} = -f_D(\hat{K}_1 + \hat{K}_2)\eta_1(\Omega_2) - f_D(\hat{K}_2)(\eta_2(\Omega_2) - \eta_1(\Omega_2)) - f_D(\hat{K}_1 + \hat{K}_2)\eta_2(\Omega_1) - f_D(\hat{K}_2)\eta_2(\Omega_{3,2}), \]

and

\[ \frac{\partial^2 \hat{\Pi}}{\partial K_1 \partial K_2} = -f_D(\hat{K}_1 + \hat{K}_2)\eta_1(\Omega_2) - f_D(\hat{K}_1 + \hat{K}_2)\eta_2(\Omega_1). \] (19)

The first order leading principle minor of \( H(\hat{\Pi}) \) is defined by \( \frac{\partial^2 \hat{\Pi}}{\partial K_i^2} \), which is non-positive given the definition of \( \eta_i(\cdot) \) and of merit order intervals \( \Omega_j \). This principle minor becomes strictly negative if \( f_D(\hat{K}_1 + \hat{K}_2) > 0 \) or \( f_D(\hat{K}_1) > 0 \).

The second order leading principle minor of the Hessian is given by its determinant, \( |H(\hat{\Pi})| \), where

\[ |H(\hat{\Pi})| = \left( \frac{\partial^2 \hat{\Pi}}{\partial K_1^2} \right) \left( \frac{\partial^2 \hat{\Pi}}{\partial K_2^2} \right) - \left( \frac{\partial^2 \hat{\Pi}}{\partial K_1 \partial K_2} \right)^2. \] (20)

Note that \( 0 \geq \frac{\partial^2 \hat{\Pi}}{\partial K_1 \partial K_2} \geq \frac{\partial^2 \hat{\Pi}}{\partial K_1^2} \), with the inequalities strict when \( f_D(\hat{K}_1) > 0 \). Also note that \( 0 \geq \frac{\partial^2 \hat{\Pi}}{\partial K_1 \partial K_2} \geq \frac{\partial^2 \hat{\Pi}}{\partial K_2^2} \), with the inequalities strict when \( f_D(\hat{K}_2) > 0 \). Therefore (20) is non-negative, and strictly positive when two or more of the following hold:

\[ f_D(\hat{K}_1) > 0, \quad f_D(\hat{K}_2) > 0, \quad \text{and} \quad f_D(\hat{K}_1 + \hat{K}_2) > 0. \] (21)

Therefore, \( \hat{\Pi}(\cdot) \) is concave in capacities \( \hat{K}_1 \) and \( \hat{K}_2 \) and strictly concave in these capacities when two or more of the conditions in (21) hold. \( \square \)

Proof of Proposition 2. Letting \( f_\varepsilon \) represent the density function for emissions price, first order conditions of (6) are symmetric and given by

\[ \frac{\partial \hat{\Pi}}{\partial K_i} = \int_{x \in \Omega_i} f_\varepsilon(x) \left[ \left( F_D(\hat{K}_1 + \hat{K}_{-i}) - F_D(\hat{K}_i) \right) (\eta_i(x) - \eta_{-i}(x)) \right] dx \\
+ \int_{x \in \Omega_i} f_\varepsilon(x) \left[ \left( 1 - F_D(\hat{K}_1 + \hat{K}_{-i}) \right) \eta_i(x) \right] dx \\
+ \int_{x \in \Omega_{-i}} f_\varepsilon(x) \left[ \left( 1 - F_D(\hat{K}_1 + \hat{K}_{-i}) \right) \eta_i(x) \right] dx \\
+ \int_{x \in \Omega_{3,i}} f_\varepsilon(x) \left[ \left( 1 - F_D(\hat{K}_i) \right) \eta_i(x) \right] dx - \beta_i = 0. \] (22)
Assume that type 1 dominates merit order in stage 2. In such a case, $\Omega_2$ and $\Omega_{3,2}$ are empty. As a result, solving (22) for total capacity, \(\{\hat{K}_1 + \hat{K}_2\}^*\), yields
\[
\{\hat{K}_1 + \hat{K}_2\}^* = F^{-1}_D \left( \frac{\bar{\eta}_2(\Omega_1) - \beta_2}{\bar{\eta}_2(\Omega_1)} \right).
\]
Substituting (23) into (22) provides the solution for type 1, the merit order dominant technology,
\[
\hat{K}^*_1 = F^{-1}_D \left( \frac{\bar{\eta}_1(\Omega_1) + \bar{\eta}_1(\Omega_{3,1}) - \beta_1 - \bar{\eta}_2(\Omega_1) + \beta_2}{\bar{\eta}_2(\Omega_1) + \bar{\eta}_1(\Omega_{3,1}) - \bar{\eta}_2(\Omega_1)} \right).
\]

The difference between (23) and (24) yields interior solutions for type 2 capacity.

Boundary conditions are obtained by the requirement that the argument in (24) be non-negative, and the requirement that the difference in the argument in (23) and (24) be non-negative.

Interior solutions and boundary conditions when type 2 dominates in the second stage are solved symmetrically. □

**Proof of Proposition 3.** Given that neither type dominates, both $\Omega_1$ and $\Omega_2$ are non-empty. Therefore, solving (22) for total installed capacity, \(\{\hat{K}_1 + \hat{K}_2\}^*\), yields
\[
\{\hat{K}_1 + \hat{K}_2\}^* = F^{-1}_D \left[ \frac{\bar{\eta}_3(\Omega_1) + \bar{\eta}_3(\Omega_{3,1}) - \beta_3 - \bar{\eta}_2(\Omega_1) + \beta_2}{\bar{\eta}_2(\Omega_1) + \bar{\eta}_1(\Omega_{3,1}) - \bar{\eta}_2(\Omega_1)} \right].
\]

Substituting (25) into (22), followed by standard algebra to isolate $\hat{K}^*_1 (\hat{K}^*_2)$ yields a solution for type 1 capacity, dependent on type 2 capacity;
\[
\hat{K}^*_1 (\hat{K}^*_2) = F^{-1}_D \left[ \left( 1 - \frac{\beta_3 - \beta_2}{\bar{\eta}_1(\Omega_1) + \bar{\eta}_1(\Omega_{3,1}) - \bar{\eta}_2(\Omega_1)} \right) \left( 1 - F_D(\hat{K}^*_2) \right) \frac{\bar{\eta}_2(\Omega_2) + \bar{\eta}_2(\Omega_{3,2}) - \bar{\eta}_1(\Omega_2)}{\bar{\eta}_2(\Omega_1) + \bar{\eta}_1(\Omega_{3,1}) - \bar{\eta}_2(\Omega_1)} \right].
\]

The solution for type 2 capacity is symmetric.

We solve for boundary conditions through the following Lemma,

**Lemma 1.** Given $\frac{\partial \bar{\eta}}{\partial K_i \partial K_{i-1}} \leq 0$, then $\frac{\partial \bar{\eta}}{\partial K_i} \big|_{K_i=0} \leq 0$ implies $\hat{K}_i^* = 0$ and $\hat{K}_i^* - \hat{K}_i = \hat{K}^*_i$.

**Proof of Lemma 1.** $\frac{\partial \bar{\eta}}{\partial K_i} \big|_{K_i=0} \leq 0$ implies $\hat{K}_i^* = 0$ through the concavity of $\bar{\Pi}(\cdot)$ in $\hat{K}_i$. Since $\frac{\partial^2 \bar{\eta}}{\partial K_i \partial K_{i-1}} \leq 0$, $\hat{K}_i^* = 0$ implies $\hat{K}_i^* - \hat{K}_i = \hat{K}^*_i$ by the definition of $\hat{K}^*_i$. □

It is evident from (19) that capacities are strategic substitutes, and therefore Lemma 1 holds.

The boundary conditions follow directly from Lemma 1 and (22). □

**Proof of Proposition 4.** The proof requires the following lemma,

**Lemma 2.** The solution set in the cap-and-trade setting is non-empty and compact.

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That the solution set is non-empty follows trivially from the zero vector as a feasible solution.

Through the negative cross-partial of $\hat{\Pi}(\cdot)$, we have established $\bar{K}^C_i$ as the effective upper bound for type $i$ capacity, $i \in \{1, 2\}$, and $0$ is the lower bound. Therefore, the solution set can be restricted to the closed and bounded Euclidian space $\{\bar{K}_1, \bar{K}_2 | 0 \leq \bar{K}_i \leq \bar{K}_i^C, \ i \in N\}$. □

Moulin (1984) shows that a Cournot-tatonnement process converges globally to the optimal solution for “nice” games (Corollary of Lemmas 1 and 2 in Moulin (1984), p 91) provided that the solution set is a non-empty and compact metric space. As shown by Lemma 2, the conditions on the solution set hold. Further, the game corresponding to the reaction functions $\hat{K}_i^*(\hat{K}_{-i})$ and $\bar{K}_i^*(\bar{K}_i)$ is “nice” in Moulin’s sense provided that the objective function is strictly quasi-concave.

The SC condition $f_{\tilde{\tau}}(x) > 0, \ \forall x \in [0, \max\{K^C_1, K^C_2\}]$ assures this. In settings in which SC does not hold, it is straightforward to approximate the original game with games that do satisfy SC whose solutions converge to the optimal solution of the original problem (e.g., subtracting $\epsilon (\bar{K}_1 + \bar{K}_2)$ from the original profit function yields an approximation which, because of the uniqueness and continuity of the resulting solution in $\epsilon$, converges to an optimal solution for the original problem as $\epsilon$ approaches $0$). □

Proof of Proposition 5. Since expected profit is clearly decreasing in $\tau$, it suffices to show the assertion when $\mu_{\bar{e}} = \tau$. Assume that capacities have been decided in stage one, but uncertainties have not yet resolved. Define $\hat{\Pi}_K$ and $\tilde{\pi}_K$ as the expected profits and expected operating margin in the cap-and-trade setting when the firm’s capacities are equal to the optimal capacity decisions in the emissions tax setting ($\Pi$ and $\pi$ continue to represent the optimal expected profit and expected operating margin in the carbon tax setting, respectively).

From (1), the second stage solution in the tax setting prior to the resolution of uncertainties is

$$
E_D[\pi] = E_D[q_{|1}(\hat{D})] \left[ p + r - b_{|1} - \alpha_{|1}\tau \right] + E_D[q_{|2}(\hat{D})] \left[ p + r - b_{|2} - \alpha_{|2}\tau \right] - rE_D[\hat{D}]
$$

(26)

Since we have assumed that $\mu_{\bar{e}} = \tau$, we can rewrite (26) as:

$$
E_D[\pi] = E_D[q_{|1}(\hat{D})] \left[ p + r - b_{|1} - \alpha_{|1}\mu_{\bar{e}} \right] + E_D[q_{|2}(\hat{D})] \left[ p + r - b_{|2} - \alpha_{|2}\mu_{\bar{e}} \right] - rE_D[\hat{D}]
$$

(27)

Define $\bar{e}_{|1}$ and $\bar{e}_{|2}$ as the emissions price threshold at which it becomes unprofitable to operate technology $[1]$ and $[2]$, $\bar{e}_{|1} = \frac{p+r-b_{|1}}{\alpha_{|1}}$ and $\bar{e}_{|2} = \frac{p+r-b_{|2}}{\alpha_{|2}}$, respectively. We can now re-write (27) as the conditional expectation

$$
E_D[\pi] = \sum_{i=1}^{2} E_D[q_{|i}(\hat{D})] \left[ Pr(\bar{e} < \bar{e}_{|i}) \left( p + r - b_{|i} - \alpha_{|i}\mu_{\bar{e}} [\bar{e} < \bar{e}_{|i}] \right) \right] + rE_D[\hat{D}]
$$

(28)
It is straightforward to show that a firm operating the optimal emission tax capacity portfolio under a cap-and-trade regime would generate expected operating margins symmetric to (28),

\[
E_{\hat{D},\tilde{e}}[\hat{\pi}_K] = \sum_{i=1}^{2} E_{\hat{D}}[\tilde{q}_i(\hat{D})] \left[ \text{Pr}(\tilde{e} < \tilde{e}_{[i]}) (p + r - b_{[i]} - \alpha_{[i]} E_{\tilde{e}}[\tilde{e} | \tilde{e} < \tilde{e}_{[i]}]) \right] + \sum_{i=1}^{2} E_{\hat{D}}[\tilde{q}_i(\hat{D})] \left[ \text{Pr}(\tilde{e} \geq \tilde{e}_{[i]}) (p + r - b_{[i]} - \alpha_{[i]} E_{\tilde{e}}[\tilde{e} | \tilde{e} \geq \tilde{e}_{[i]}]) \right] - r E_{\hat{D}}[\hat{D}]
\]

(29)

When \( p + r - b_i - \alpha_i e \leq 0 \), then technology \( i \in N \setminus \bar{N} \), and \( \tilde{q}_i = 0 \) by (5). Therefore, (29) becomes

\[
E_{\hat{D},\tilde{e}}[\hat{\pi}_K] = \sum_{i=1}^{2} E_{\hat{D}}[\tilde{q}_i(\hat{D})] \left[ \text{Pr}(\tilde{e} < \tilde{e}_{[i]}) (p + r - b_{[i]} - \alpha_{[i]} E_{\tilde{e}}[\tilde{e} | \tilde{e} < \tilde{e}_{[i]}]) \right] - r E_{\hat{D}}[\hat{D}]
\]

(30)

When \( p + r - b_i - \alpha_i e > 0 \), then technology \( i \in \bar{N} \) and \( \tilde{q}_i = q_i \) by (5) and (2). As a consequence, by (28) and (30), \( E_{\hat{D},\tilde{e}}[\hat{\pi}_K] \geq E_{\hat{D}}[\pi] \) is equivalent to

\[
0 \geq \sum_{i=1}^{2} E_{\hat{D}}[q_i(\hat{D})] \left[ \text{Pr}(\tilde{e} \geq \tilde{e}_{[i]}) (p + r - b_{[i]} - \alpha_{[i]} E_{\tilde{e}}[\tilde{e} | \tilde{e} \geq \tilde{e}_{[i]}]) \right],
\]

which holds always, and holds strictly if \( \text{Pr}(\tilde{e} \geq \tilde{e}_{[2]}) > 0 \) for interior solutions, or if \( \text{Pr}(\tilde{e} \geq \tilde{e}_{[1]}) > 0 \) for boundary solutions. Given that \( E_{\hat{D},\tilde{e}}[\hat{\pi}_K] \) and \( E_{\hat{D}}[\pi] \) are derived from identical capacity portfolios, investments costs are identical, so \( \hat{\Pi}_K(\hat{D},\tilde{e}) \geq \Pi(\hat{D},\tilde{e}) \). Further, \( \hat{\Pi}(\hat{D},\tilde{e}) \geq \hat{\Pi}_K(\hat{D},\tilde{e}) \) through profit maximization. Therefore, \( \hat{\Pi}(\hat{D},\tilde{e}) \geq \Pi(\hat{D},\tau) \). \( \square \)

**Appendix 2: Numerical Experiment Context**

The setting described to motivate the numerical experiment in Section 5 derives from the comparison of current kiln technology within the cement industry to cost and efficiency projections for CCS technology. Although CCS technology will not be commercially available within the industry until 2020, a firm planning potential capacity investments today would be interested in such a comparison, with the result potentially impacting the firm’s decision to retire installed capacity, replacing it with current technology (which has an expected life of 50 years), or to invest in extending the life of installed capacity in order to eventually replace it with CCS technology.

Traditional kiln parameters for the numerics are derived from a combination of financial statements from a large European cement manufacturer and conversations with industry executives. CCS projections come from the European Cement Research Academy’s Cement Sustainability Initiative report on Oxyfuel technology (Cement Sustainability Initiative 2009) and conversations with industry executives.
Investment costs ($\beta_1 = 10.1, \beta_2 = 14.6$): Investment costs were derived to offset capital expenses of 200 million euro and 290 million euro for standard and CCS kilns, assuming a discount rate of 10%, an expected life of 50 years, and an annual production of 2 million tons. Within Figures 1a and 1b, the range for $\beta_2$ represents a maximum decrease of 39.7 million euro from the CCS kiln capital expense up to a maximum increase of 19.8 million. Decreases could result from technology improvements, government subsidy, or technology producer price discounts.

Operating costs ($b_1 = 43.6, b_2 = 55.0$): Operating costs for standard kilns, $b_1$, come from the annual report of a large European cement manufacturer and include estimated material, fuel and labor costs per ton of cement produced by standard kilns. CCS non-emissions operating costs, $b_2$, add the 8 euro estimated operating cost increase indicated by the Cement Sustainability Initiative paper (Cement Sustainability Initiative 2009) as well as the cost for the additional estimated 88kWh of energy required in the CCS case (with the latter derived from the estimated 110 kWh per ton of clinker increase in electricity requirements times 0.80 tons of clinker per ton of cement, at a cost of 0.0385 euro per kWh).

Technology emissions intensity ($\alpha_1 = 0.700, \alpha_2 = 0.075$): The emissions intensity for standard kilns, $\alpha_1 = 0.700$, represents a rough average for the industry’s gross emissions per ton. Gross emissions include the emissions generated from alternative fuels, which are currently included within the EU-ETS assessment of a facility’s emissions. The emissions intensity for CCS kilns incorporates the estimated capture efficiency of greater than 85% (Cement Sustainability Initiative 2009). The range of emissions intensity values included within Figure 2a and 2b are achievable through the use of clinker substitutes such as blast furnace slag and fly ash (Cement Sustainability Initiative 2009).

Price ($p = 90$): Revenue per ton of cement assumes a price today of approximately 74 euros per ton of cement (estimated as the average price in Europe during 2008 based on a major cement manufacturer’s financial statements), and a 2% increase in price per ton between 2011 through 2020, when CCS technology is expected to become available.

Emissions allowance price distribution: (lognormal, $\mu_e = 25$, and $\sigma_e = 15$): The mean of the emissions allowance price distribution roughly reflects the average EU-ETS Phase III projections from eight banks or carbon allowance analysts (an average estimate of 25.9 euros per ton based on projections by Barclays Capital, Citibank, Daiwa, Deutsche Bank, Point Carbon, Sagacarbon, Société Générale, and UBS) as presented by the focal firm. A standard deviation of 15 ($\sigma_e = 0.60$) was used to reflect the considerable uncertainty in future allowance pricing.

Demand distribution (lognormal, $\mu_D = 1000$, and $\sigma_D = 175$): The demand distribution’s mean was taken from the average European sales volume reported in the focal firm’s recent financial
statements. This mean was then normalized to 1,000 units while preserving the focal firm’s estimated coefficient of variation. Coefficient of variation, $\frac{\sigma_{D}}{\mu_{D}} = 0.175$, was estimated loosely, with a reported “down year” for the firm’s cement business in Europe assumed to be one standard deviation below the mean annual demand. Normalizing to a mean demand of 1,000 was done both to preserve confidentiality and to simplify projections to other substantial cement producers or to the region as a whole.

Appendix 3: Additional Numerical Results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$K_1$</th>
<th>$K_2$</th>
<th>Total $K_2$ Share</th>
<th>$\psi_2$</th>
<th>Profit $\Pi$</th>
<th>Emissions $\epsilon$</th>
<th>Intensity $\nu_q$</th>
<th>Production $\varphi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_1$</td>
<td>-3.56</td>
<td>28.18</td>
<td>0.00</td>
<td>28.18</td>
<td>-0.54</td>
<td>-2.94</td>
<td>-3.02</td>
<td>0.07</td>
</tr>
<tr>
<td>$b_1$</td>
<td>-20.76</td>
<td>167.22</td>
<td>0.34</td>
<td>166.31</td>
<td>-1.98</td>
<td>-18.89</td>
<td>-19.30</td>
<td>0.51</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>-5.54</td>
<td>45.15</td>
<td>0.15</td>
<td>44.94</td>
<td>-0.78</td>
<td>-3.90</td>
<td>-3.99</td>
<td>0.10</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>5.13</td>
<td>-42.77</td>
<td>-0.25</td>
<td>-42.62</td>
<td>-0.08</td>
<td>4.15</td>
<td>4.36</td>
<td>-0.21</td>
</tr>
<tr>
<td>$b_2$</td>
<td>12.02</td>
<td>-100.00</td>
<td>-0.56</td>
<td>-100.00</td>
<td>-0.12</td>
<td>9.37</td>
<td>9.89</td>
<td>-0.48</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>0.60</td>
<td>-5.02</td>
<td>-0.03</td>
<td>-4.99</td>
<td>-0.01</td>
<td>0.52</td>
<td>0.54</td>
<td>-0.02</td>
</tr>
<tr>
<td>$\mu_{\hat{\epsilon}}$</td>
<td>-0.63</td>
<td>9.09</td>
<td>0.52</td>
<td>9.12</td>
<td>-0.93</td>
<td>-0.56</td>
<td>-0.92</td>
<td>0.37</td>
</tr>
<tr>
<td>$\sigma_{\hat{\epsilon}}$</td>
<td>0.10</td>
<td>-0.70</td>
<td>0.01</td>
<td>-0.71</td>
<td>0.06</td>
<td>0.03</td>
<td>0.09</td>
<td>-0.06</td>
</tr>
</tbody>
</table>

Table A3.1 Elasticity of metrics to parameters under cap-and-trade.

<table>
<thead>
<tr>
<th>Change in $\beta_2$</th>
<th>$K_1$</th>
<th>$K_2$</th>
<th>Total Capacity $\psi_2$</th>
<th>Profit $\Pi$</th>
<th>Emissions $\epsilon$</th>
<th>Intensity $\nu_q$</th>
<th>Output $\varphi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2.00</td>
<td>179</td>
<td>891</td>
<td>1,070</td>
<td>83.3%</td>
<td>18.452</td>
<td>149</td>
<td>0.156</td>
</tr>
<tr>
<td>-1.75</td>
<td>201</td>
<td>869</td>
<td>1,070</td>
<td>81.2%</td>
<td>18.231</td>
<td>160</td>
<td>0.168</td>
</tr>
<tr>
<td>-1.50</td>
<td>226</td>
<td>844</td>
<td>1,070</td>
<td>78.9%</td>
<td>18.017</td>
<td>173</td>
<td>0.182</td>
</tr>
<tr>
<td>-1.25</td>
<td>254</td>
<td>817</td>
<td>1,070</td>
<td>76.3%</td>
<td>17.809</td>
<td>189</td>
<td>0.198</td>
</tr>
<tr>
<td>-1.00</td>
<td>288</td>
<td>783</td>
<td>1,070</td>
<td>73.1%</td>
<td>17.609</td>
<td>208</td>
<td>0.218</td>
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<tr>
<td>-0.75</td>
<td>336</td>
<td>734</td>
<td>1,070</td>
<td>68.6%</td>
<td>17.419</td>
<td>236</td>
<td>0.248</td>
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<tr>
<td>-0.50</td>
<td>601</td>
<td>469</td>
<td>1,070</td>
<td>43.8%</td>
<td>17.250</td>
<td>397</td>
<td>0.419</td>
</tr>
<tr>
<td>-0.35</td>
<td>807</td>
<td>260</td>
<td>1,067</td>
<td>24.4%</td>
<td>17.203</td>
<td>522</td>
<td>0.553</td>
</tr>
<tr>
<td>-0.275</td>
<td>842</td>
<td>224</td>
<td>1,066</td>
<td>21.0%</td>
<td>17.185</td>
<td>542</td>
<td>0.576</td>
</tr>
<tr>
<td>-0.25</td>
<td>853</td>
<td>213</td>
<td>1,065</td>
<td>20.0%</td>
<td>17.179</td>
<td>548</td>
<td>0.583</td>
</tr>
<tr>
<td>0</td>
<td>942</td>
<td>119</td>
<td>1,061</td>
<td>11.2%</td>
<td>17.138</td>
<td>597</td>
<td>0.637</td>
</tr>
<tr>
<td>0.25</td>
<td>1,026</td>
<td>30</td>
<td>1,056</td>
<td>2.9%</td>
<td>17.119</td>
<td>640</td>
<td>0.685</td>
</tr>
<tr>
<td>0.5</td>
<td>1,055</td>
<td>0</td>
<td>1,055</td>
<td>0.0%</td>
<td>17.118</td>
<td>653</td>
<td>0.700</td>
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Table A3.2a Results from the cap-and-trade setting when changing $\beta_2$. 

### Table A3.2b Results from the emissions tax setting when changing $\beta_2$.

<table>
<thead>
<tr>
<th>Change in $\beta_2$</th>
<th>Capacity Metrics</th>
<th>Performance Metrics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$K_1$, $K_2$</td>
<td>Total $K_2$ share</td>
</tr>
<tr>
<td></td>
<td>Capacity $\psi_2$</td>
<td></td>
</tr>
<tr>
<td>-2.00</td>
<td>107 946 1,053 89.8%</td>
<td>18,342 103 0.108 953</td>
</tr>
<tr>
<td>-1.25</td>
<td>187 867 1,053 82.3%</td>
<td>17,661 137 0.144 953</td>
</tr>
<tr>
<td>-0.50</td>
<td>309 744 1,053 70.6%</td>
<td>17,050 203 0.214 953</td>
</tr>
<tr>
<td>-0.25</td>
<td>787 266 1,053 25.3%</td>
<td>16,894 501 0.525 953</td>
</tr>
<tr>
<td>0.00</td>
<td>1,053 0 1,053 0.0%</td>
<td>16,894 667 0.700 953</td>
</tr>
<tr>
<td>0.50</td>
<td>1,053 0 1,053 0.0%</td>
<td>16,894 667 0.700 953</td>
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### Table A3.3a Results from the cap-and-trade setting when changing $\alpha_1$.

<table>
<thead>
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<th>$\alpha_1$</th>
<th>Capacity Metrics</th>
<th>Performance Metrics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$K_1$, $K_2$</td>
<td>Total $K_2$ share</td>
</tr>
<tr>
<td></td>
<td>Capacity $\psi_2$</td>
<td></td>
</tr>
<tr>
<td>0.650</td>
<td>1,062 0 1,062 0.0%</td>
<td>18,239 612 0.650 941</td>
</tr>
<tr>
<td>0.670</td>
<td>953 112 1,065 10.5%</td>
<td>17,807 580 0.616 942</td>
</tr>
<tr>
<td>0.685</td>
<td>847 222 1,068 20.8%</td>
<td>17,522 535 0.567 944</td>
</tr>
<tr>
<td>0.700</td>
<td>389 681 1,070 63.7%</td>
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<tr>
<td>0.720</td>
<td>279 787 1,066 73.8%</td>
<td>17,202 207 0.217 953</td>
</tr>
<tr>
<td>0.740</td>
<td>229 834 1,062 78.5%</td>
<td>17,123 180 0.189 952</td>
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### Table A3.3b Results from the emissions tax setting when changing $\alpha_1$.

<table>
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<th>$\alpha_1$</th>
<th>Capacity Metrics</th>
<th>Performance Metrics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$K_1$, $K_2$</td>
<td>Total $K_2$ share</td>
</tr>
<tr>
<td></td>
<td>Capacity $\psi_2$</td>
<td></td>
</tr>
<tr>
<td>0.650</td>
<td>1,061 0 1,061 0.0%</td>
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</tr>
<tr>
<td>0.670</td>
<td>1,058 0 1,058 0.0%</td>
<td>17,609 639 0.670 954</td>
</tr>
<tr>
<td>0.685</td>
<td>1,056 0 1,056 0.0%</td>
<td>17,251 653 0.685 954</td>
</tr>
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<td>0.700</td>
<td>285 769 1,053 73.0%</td>
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<tr>
<td>0.740</td>
<td>169 878 1,047 83.8%</td>
<td>16,995 134 0.141 951</td>
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</table>